Analysis of Marked Graph of S^4PR Model of Flexible Manufacturing System Using Sign Incidence Matrix

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Abstract:

In this paper we take a marked graph of FMS OF S^4PR from [6] and analyze it using sign incidence matrix suggested in [3][4][5].

***Keywords**:* Flexible Manufacturing Systems, Petri Nets, Marked Graphs, Sign Incidence Matrix systems of simple sequential process with shared resources (S^4PR)

I. INTRODUCTION:

Petri nets are introduced in [1][2]. Generating basis siphons and traps of Petri nets using the sign incidence matrix discussed in [3][4][5]. We take the marked graph of S^4PR model of FMS from [6] and analyze it using sign incidence matrix suggested in [3][4][5]. Rest of the paper is organized as follows section. I contain basic definitions of Petri Nets. Section II contains the algorithm contained in [3][4][5]. Section III contains the analysis of marked graph of S^4PR model of FMS taken from [6]. Section V contains conclusion and references.

A Basic Definitions:

Definition 1.1. A PN is a bipartite graph, where nodes are classified as places and transitions (graphically pictured as circles and bars, respectively), and directed arcs connect only nodes of different type. Places are endowed with integer variables called tokens. More formally, a marked PN is a 5-tuple $N = (P, T, F, W, M_0)$, where P is a finite set of places, T is a finite set of transitions, with $P \cap T = \emptyset, F \subset (P x T) U (T x P)$ is the incidence or flow relation (each element of F corresponds to an arc in the PN), $W : F \longrightarrow N \setminus \{0\}$ is the arc weight function, and $M_0: P \longrightarrow N$ is the initial marking (a marking $M : P \longrightarrow N$ defines the distribution of tokens in places), where N is the set of natural numbers.

Definition1.2: **Marked graph.**

A Marked graph is a Petri net in which each place as exactly one input transition and one output transition.

Definition 2.1: For a Petri net N with n-transitions and m-places, the sign incidence matrix $A = [a_{ij}]$ is an n x m matrix whose entry is given as follows .

a_{ij} = + if place j is an output place of transition i.

a_{ij} = — if it is an input place of transition i.

a_{ij} = ± if it is both input and output places of transition i (i.e. transition i and place j form a self loop and

a_{ij} = 0 otherwise.

Definition 2.2 : The addition denoted by $\oplus$ is a commutative binary operation on the set of four element

B = {+,-, 0, ± } defined as follows.

$+ \oplus - = \pm$

$x \oplus x = x$, For every $x \in B$

$\pm \oplus x = \pm$, For every $x \in B$

$0 \oplus x = x$, For every $x \in B$

Definition:2.3

A subset of places denoted as Z is both siphon and trap if $Z^* = {}^*Z$

II. Enumeration of siphon and trap as subsets of places of marked graphs:

Here we present an algorithm given in [4] for marked graphs to find all subsets of places which are both siphon and trap. We define a siphon-trap matrix for marked graphs. A relation between sign incidence matrix and siphon-trap matrix for marked graphs is obtained.

Theorem 2.4: A subset of k-places Z = {p_1, p_2> ...... p_k) in a marked graph N is

both siphon and trap iff the addition of k-column vectors of the sign incidence matrix of N, $A_1 \oplus A_2 \oplus ..... \oplus A_k$ contains either zero entry or ± entry where A_j

Denote the column vector corresponding to place P_j, j = 1,2,....k.

Proof: Let $A_1 \oplus A_2 \oplus .... \oplus A_k = V = [v_j]$, where Vj denote the ith row of the column vector V. The following statements are obvious from the definition of sign incidence matrix and the operation ⊕.

(a) V_i = 0 means that no place in Z is an input or output place of

transition i.

(b) V_j = + means that some place in Z is an output place of transition i.

(c) v_i = -means that some place in Z is an input place of transition i. and

(d) V_i = ± means that some place in Z is an input place of transition i and some place in Z is an output place of transition i.

Therefore it can be seen that every transition having an output place in Z has an input place in Z if and only if $V_i \neq +$, for every i and every transition having an input place in Z has an output place in Z iff $V_i \neq$ — for every i. Thus every transition having an input place in Z and an output place in Z iff $V_i \neq$ + or —. That is if the vector V has only either zero entry or ± entry. Thus Z is both siphon and trap iff the vector V contains only either zero or ± entries.

Definition2.5: A + entry is said to be neutralized by adding a—entry to get a ± entry.

B ALGORITHM 2.6 :

Input : Sign incidence matrix A of order m x n.

Step 1 Select A_j, the first column in the sign incidence matrix A, whose corresponding place is denoted as $PLACE_j$.

Set recursion level r to 1

Set $V_{jr} = A_j$

Set $PLACE_{j\,r} = PLACE_j$

Step 2 If V_{jr} has a ± entry at i^{th} row then $PLACE_j$ is a self loop with transition t_j. Go to step 5.

Step 3 If $V_{i\,j}$ has a + entry in the k^{th} row find a column in A, which contains a

- entry at the k^{th} row.

(a) If no such column in $A_,$ exists, Go to step 5.

(b) If such A, exists add it to $V_{j\,r}$ to obtain $V_{j(r+1),} = V_{j\,r} \oplus A_s$ containing a ± entry at k^{th} row. Then $PLACE_{j(r+1)} = PLACE_{j\,r}$ U $PLACE_s$.

(c) Repeat this step for all possible nutralizing columns A_s. This gives a new set of $V_{j(r+1)}$'s and $PLACE_{j(r+!)}$'s.

Step 4 Increment r by 1. Repeat step 3 until there are no more + entries in each

$V_{jr} = A_1 \oplus A_2 \oplus$ ...A_{jr} or no neutralizing column can be defined .

Step 5 Any V_{jr} without + entries and without-entries (i.e., all the entries are either zero or ±) represents siphon and trap (By theorem). i.e., the places in $PLACE_{j\,r}$ form both siphon and trap.

Step 6

Delete A_j

j=j+1

Go to step 1.

Output : All sets which are both siphon and trap.

Definition 3.1

S^4 PR nets- . Let I_N = { 1, 2, ..., *m}* be a finite set of indices. A well-marked S^4PR net is a generalized self-loop free Petri net *N = (P, T, W, m_0),* where

i) $P = P_S \cup P_0 \cup P_R$ is a partition such that $P_{sI} = \cup i \in I_N P_{Si}$, $P_{sI} \neq \emptyset$ $PS_I \cap PS_I = \emptyset$, $\forall i \neq j$ (state places), $\cup i \in I_N v\{P_0\}$ (idle places), P_R = {r,, r_2, ..., r„},n>0 (resource places);

ii) $T = i \in I_N T_{i,}$ $Ti \neq \emptyset$ *and* $T_i \cap T_j = \emptyset$, $\forall i \neq j$;

iii) $\forall i \in I_N$ the sub-net generated by $P_{s1} \cup \{P0_I\} \cup T_i$ strongly connected state machine, such that every contains P_0;

iv)) $\forall r \in P_R$, $\exists$ A unique P-invariant x,. such that $\{r\} = ||x_r|| \cap P_R$, $P_0 \cap ||x_r|| \neq \emptyset$ and $x_r(r) = 1$;

v) $P_s = \cup_{r \in P_R} H_r$ where $H_r = ||x_r|| \setminus \{r\}$;

vi) N is a connected net;

vii) $m0(p)=0$, $\forall p \in P_s$, $m0(r) \geq \max_{p \in ||x_r||} x_r(p)$, $\forall r \in P_R$, $m0(p) \geq 0$, $\forall p \in P0_j \in P0$.

An S4PR net is covered by non-negative P-invariants (related to resources and idle places) and is therefore conservative and bounded. The set *H,.* denotes the set of (state places) holders of resource *r.* Notice that, since in an S4PR net the production sequences are modeled as state machines, a firing sequence related to a process will end up marking again the respective idle place, unless a deadlock occurs (production sequences are repeatable). The marking of the idle places allows the activation of the state machines, so that deadlock can occur only because of a disabling resource place in a deadly marked siphon. |

In the following, it will be assumed that the sequencing of the operations in the process, as well as the resource allocation mechanism, which constitute the core of the supervisor, are modeled by a suitable S4PR. The proposed DP method further constrains the model behavior to avoid unwanted, reachable states. The final supervisor is then obtained by suitably associating the PN transitions to input/output events in order to communicate with the plant.

C The RADP approach

Deadlock classification based on resource partition

Each deadlock state can be characterized in terms of the resources that are involved in a circular wait condition. The associated resource places are empty in the corresponding deadly marked siphons, so that siphons can be characterized in terms of the resources as well.

The RADP approach designs the control for the deadlock states in different ways according to the specific resources involved. More specifically, let the resource set be partitioned in two arbitrary non-empty sets $P_{AR} \cup P_{UR}$, $P_{AR} \cap P_{UR} = \emptyset$. where P_{AR} groups resources that will be booked *before* actual use (anticipated booking) and P_{UR} the remaining resources (unanticipated booking), briefly referred to as A-resources and U-resources, respectively, Partitioning criteria will be discussed in Sect. 6. Siphons and deadlocks can be categorized as follows depending on the involved resources:

Definition 3.3 - A-, U- and M- siphons and deadlocks Let *N = (P, T, W, mo)* be a well-marked S4PR net. Let $P_R = P_{AR} \cup P_{UR}$ and II the set of minimal siphons of *N.* II can be partitioned as follows:

$\Pi = \Pi A \cup \Pi U\ \Pi M$. $\Pi A\ \Pi U = \Pi A \cap \Pi M$

$= \Pi U\ \Pi M = \emptyset$,

Where

i) The set $\Pi A = \{S |\ S \cap PUR = \emptyset\}$ Of A-siphons contains siphons involving only A-resources.

ii) The set $\Pi U = \{S |\ S \cap PRR = \emptyset\}$ Of A-siphons contains siphons involving only U-resources

iii) The set $\Pi M = \Pi \setminus (\Pi A \cup \Pi U)$ of M-siphons (mixed siphons) contains involving both A- and U- resources.

Deadlocks associated to deadly marked *A-, U-* and *M-* siphons are called *A-, U-* and M-deadlocks.

In order to decouple the DP problem, the overall model N must be modified by suitably anticipating the allocation of y4-resources as described in the following.

Anticipated booking model Let N = (P, T, W, m_0) be a well-marked S4PR net. Let $PR = P_{AR} \cup P_{UR}$ and $HUR = \cup_{r \in P_{UR}}$. For each resource place $r \in P_{AR}$ perform the following algorithm:

While True

Let $T_{ar} = r\bullet$.

Let P_{ar}. = $(\bullet T_{ar}) \cap H_{UR}$

If $P_{ar} = \emptyset$ then Exit, End If.

While $P_{ar} = \emptyset$

Choose $p \in P_{ar}$.

Let $w = W_{(r,t)}$, where $t \in T_{ar} \cap (p\bullet)$.

Set $W_{(r,ta)} = W_{(r,ta)} + w \quad \forall t_a \in \bullet p$.

Set $W_{(t_p,r)} = W_{(t_p,r)} + w \quad \forall t_P \in (\bullet p) \setminus T_{ar}$

Set $W_{(r,t)} = W_{(r,t)} - w \quad \forall t \in T_{ar} \cap (p\bullet)$.

Set $P_{ar} -= P_{ar} \setminus \{p\}$.

End While

End While

The resulting model is denoted N_{AB} and termed anticipated booking model (AB-model).

The rule applied in Def. 3 includes place *p* in ||xr|| (p becomes a holder of r) and the net topology is suitably adjusted to account for the modified resource P-invariant. Some examples of the effect of the resource anticipation rule are given in Fig.1

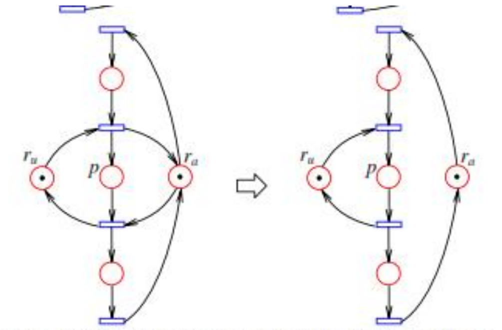

Fig. 1 Examples of the resource anticipation mechanism used in Def. 3, where $P_{AR} = \{r_a\}$, $P_{UR} = \{r_u\}$ and the idle place is omitted for brevity

We name the Places and transitions of the above Petri net as follows

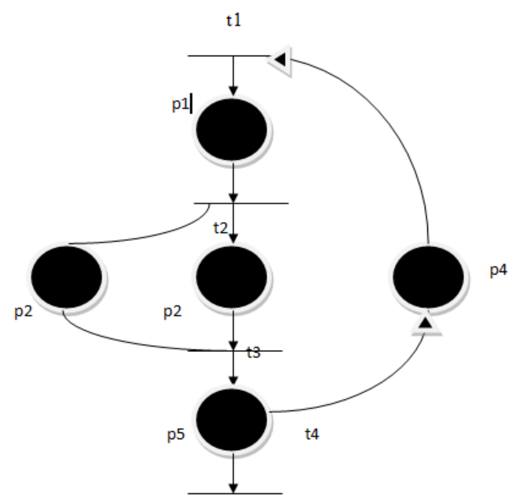

FIG 2

The sign incidence matrix for the above matrix is given by

$$A = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \end{array} \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \\ + & 0 & 0 & - & 0 \\ - & - & + & 0 & 0 \\ 0 & + & - & 0 & + \\ 0 & 0 & 0 & + & - \end{bmatrix}$$

$V_{11} = A_1 = \begin{bmatrix} + \\ - \\ 0 \\ 0 \end{bmatrix}$.**V_{11}** *has a + in Ist row.* The neutralizing column

is A_4 Place $_{11}$ = $\{p_1\}$

$$V_{12}(1) = V_{11}+A_4 = \begin{bmatrix} + \\ - \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} - \\ 0 \\ 0 \\ + \end{bmatrix} = \begin{bmatrix} \pm \\ - \\ 0 \\ + \end{bmatrix}$$

PLACE $_{12}^{(1)} = \{p_1, p_4\}$

$V_{12}^{(1)}$ has a + in 4th row. The neutralizing column is A_5

$$V_{13}^{(1)} = v_{12}^{(1)} + A_5 = \begin{bmatrix} \pm \\ - \\ 0 \\ + \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ + \\ - \end{bmatrix} = \begin{bmatrix} \pm \\ - \\ + \\ \pm \end{bmatrix}$$

PLACE $_{13}^{(1)} = \{p_1, p_4, p_5\}$

$V_{13}^{(1)}$ HAS a + IN 3RD row. The neutralizing column is A_3.

$$\text{Therefore } V_{14}^{(1)} = V_{13}^{(1)} + A_3 = \begin{bmatrix} \pm \\ - \\ + \\ \pm \end{bmatrix} + \begin{bmatrix} 0 \\ + \\ - \\ 0 \end{bmatrix}$$

$$\text{PLACE}_{14}^{(1)} = \{p_1, p_4, p_5, p_3,\} = \begin{bmatrix} \pm \\ \pm \\ \pm \\ \pm \end{bmatrix}$$

Since all entries is in $V_{14}^{(1)}$ are ± the set of places $\{p_1, p_4, p_5, p_3,\}$

forms siphon and trap.

Similar analyzing other columns we get these sets as siphon and traps.

$Z_2 = \{p_2, p_{3,}\}$, $Z_3 = \{p_3, p_{1,}, p_2\}$, $Z_4 = \{p_3, p_{1,,} p_4, p_{5,}\}$

Therefore the siphon trap matrix of the given marked graph is

$$\begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array}\begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

By converting the given marked graph into a directed graph by transforming transitions into vertices and places into edges we get the following directed graph.

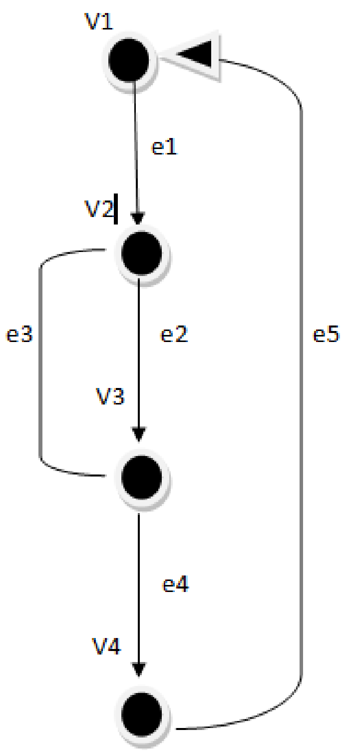

FIG 3

This graph is not an Euler digraph for the vertices V_2, V_3, in degree is not equal to out degree. Therefore for the analysis is not possible as per [4][5]

III. CONCLUSION

In this paper we found the set of places which are both siphon and trap of the marked graph of FMS of S[4]PR . We converted the marked graph into a directed graph. We found that its not a Euler diagraph. Therefore further analysis according to [4][5] is not possible

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