TOTAL SEQUENTIAL CORDIAL LABELING OF UNDIRECTED GRAPHS

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Abstract

A graph with V as vertex set and E as edge set is said to have Total Sequential Cordial labeling(TSC), if there exists a mapping $f : V \cup E \rightarrow \{0,1\}$ such that for each $(a,b) \in E$, $f(ab) = |f(a) - f(b)|$, provided the condition $|f(0) - f(1)| \leq 1$ is hold, where $f(0) = v_f(0) + e_f(0)$ and $f(1) = v_f(1) + e_f(1)$ and $v_f(i)$, $e_f(i)$, $i \in \{0,1\}$ are respectively, the number of vertices and edges labelled with i. In this paper we study Total Sequential Cordial Labeling for some undirected graphs.

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I. INTRODUCTION

All graphs considered are finite, simple and undirected. The vertex set and edge set of a graph G is denoted by V(G) and E(G) respectively. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

The graph labeling problem was first introduced by Alex Rosa in the year 1967.Based on this many graph labeling problems have been defined and introduced as graceful, Harmonious, felicitous, elegant, cordial, magic anti magic, bimagic and prime labeling etc. A detailed history of graph labeling problems and related results are presented by Gallian [1].The graph labelling problem that appears in graph theory has a fast development recently. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance x-rays crystallography, coding theory, radar, astronomy, circuit design, and design of good Radar Type Codes, Missile Guidance Codes and Convolution Codes with optimal autocorrelation properties and communication design.

II. RELATED SURVEY

Two of the most important types of labeling are called graceful and harmonious labelling. Graceful labeling were introduced by Rosa [9] in 1996 and Golomb [7] in 1972. Harmonious labelling were first studied by Graham and Sloane [8] in 1980.A third type of labeling is cordial labeling and was introduced by Cahit [2] in 1987.After Cahit the meaning of *cordiality* in the graph labeling problems is well understood and studied [3], [4].Cahit also introduced total magic cordial and total sequential cordial labeling in 2002[5].The notion of total magic cordial labeling and total sequential cordial labeling was a modification of edge magic and cordial labeling, sequential and cordial labelling respectively. Cahit defined total sequential cordial labeling as a weaker version of simply sequential labeling of graphs.He proved that every cordial graph is TSC, Cn is TSC for all n > 2, trees are TSC, the wheel Wn is TSC for all n > 3. He gave some conditions for a complete graph Kn to be TSC.

C.Nirmalakumari and T.Nicholas[6]has proved that the graphs Cn, and Pn are TSC graphs. Moreover, they show that the join of the path P_n and the star $K_{1,m}$ is TSC if and only if n ≠ 2 and m is even; the union of the path P_n and the star $K_{1,m}$ is total cordial if and only if n ≠ 2 and m is even. In this paper we proved that the path related graph P_n^2 and the shadow graphs of path and star are total sequential cordial graphs.

III. PRELIMINARIES

We will give brief summary of definitions which are useful for the present investigations.

*Definition 2.1:*Let G (V,E) be an undirected graph. A labeling is called *Total Sequential Cordial labeling*, if there exists a mapping f : V∪E→{0,1} such that for each (a,b)∈E, f(ab) =|f(a) – f(b)| , provided the condition |f(0) – f(1)|≤1 is hold, where f(0) = $v_f(0)$ + $e_f(0)$ and f(1) = $v_f(1)$ +$e_f(1)$ and v_f (i) , $e_f(i)$, i∈{0,1} are respectively, the number of vertices and edges labelled with i.

*Definition 2.2:*Let f : V→ {0,1}and for each edge xy assign the label |f(x)-f(y)|. Call f a *Cordial Labeling* of G if the number of vertices labelled 0 and the number of vertices labelled 1 differs by at most 1 and the number of edges labelled 0 and the number of edges labelled 1 differs by at most 1.

Definition 2.3: A graph G (V,E) is called *simply sequential* if there is a bijection f: V∪E → {1,2,…,|V| +|E|} such that for each edge (a,b) ∈E, f(ab) = | f(a) – f(b)|.

*Definition 2.4:*The *Shadow graph*$D_2(G)$ of aconnected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

*Definition 2.5:*A *path* is sequence of edges which connects a sequence of vertices. If the start and end vertices are the same then it is called closed path, and if they are not same then it is an open path.

*Definition2.6:*The k^{th} power G^k of a connected graph G, where k≥1, is that graph with V (G^k) = V(G) for which uv∈ E(G^k) if and only if 1≤ dG (u,v) ≤ k .The graphs G^2 and G^3 are also referred to as square and cube respectively of G.

*Definition2.7:*The *star graph* S_n of order n, sometimes simply known as n -star, is a tree on n nodes with one node having vertex degree n-1 and the other n-1 having vertex degree 1.

IV. MAIN RESULT

In this section we show the existence of Total Sequential Cordial labeling for P_n^2, shadow graph of path and staralso present an algorithm to get the total sequential cordial labeling for the same graphs.

3.1: Total Sequential Cordial Labeling Of P_n^2

Algorithm 3. 1.1:

Step 1: Let Vbe the set of vertices
whereV = {v_1, v_2, ...,v_n}
Step 2: Let E_1 and E_2 be the set of edges
where E_1 = {v_1v_2, v_2v_3, ...,$v_{n-1}v_n$},
E_2 = {v_1v_3,v_2v_4,v_3v_5,…$v_{n-2}v_n$}
Step 3: Define f: V∪E → {0, 1 } as
Display

$$f(v_i) = \begin{cases} 0 & i \quad odd \\ 1 & i \quad even \end{cases}$$

$$f(v_i\, v_{i+1}) = 1 \qquad 1 \le i \le n-1$$

$$f(v_i\, v_{i+2}) = 0 \qquad 1 \le i \le n-2$$

end

f_0=no. of vertices and edges labelled 0;
f_1=no. of vertices and edges labelled 1;
display $|f_0-f_1|$

end.

Theorem3.1.2: The Path related graph P_n^2 admits total sequential cordial labeling.

Proof: Let P_n^2 be a graph with n vertices and 2n-3 edges. The edge set and vertex set are explained in the above algorithm. To prove the Path related graph P_n^2 for every n, admits Total sequential cordial labeling, we define a mapping f: $V \cup E \rightarrow \{0,1\}$ such that for all $(a,b) \in E$, $f(ab) = |f(a) - f(b)|$,as defined in step 3 of the above algorithm.

Let f_0 be the number of vertices and edges labelled zero and f_1 be the number of vertices and edges labelled one.

Case1: When n is even

f_o= [n/2+n-2]

f_1 = [n/2+n-1]

then $|f_o - f_1| = |(\frac{3n}{2}) - 2 - \left(\frac{3n}{2}\right) + 1| = 1.$

Case2: When n is odd

f_o=[((n+1)/2+n-2]

f_1 = [(n-1)/2+ n-1]

then $|f_o - f_1| = |(\frac{3n-3}{2}) - (\frac{3n-3}{2})| = 0.$

It shows that the number of vertices and edges labelled zero and number of vertices and edges labelled one differ at most by one in both cases. Hence the graph Pn^2 admits total sequential cordial labeling.

3.2: Total Sequential Cordial Labeling Of Shadow Graphs $D_2(P_n)$ and $D_2(K_1,n)$

The total sequential cordial labeling of Shadow graph $D_2(P_n)$given in the following algorithm.

Algorithm:3.2.1:

Step 1: Let V, V□ be the set of vertices where V = {v_1, v_2, ...,v_n} and V□ = { v_1□,v_2□,...,v_n□}

Step 2: Let E_1, E_2, $E_3$and E_4 be the set of edges where E_1 = {v_1v_2, v_2v_3, ...,$v_{n-1}v_n$},

E_2 = {v_1v_2□, v_2v_3□,…,$v_{n-1}v_n$□},

E_3 = {v_1□v_2□,v_2□v_3□,…,v_{n-1}□v_n□} and

E_4 = {v_1□v_2,v_2□v_3,…,v_{n-1}□v_n}

Step 3: Define f: $V \cup E \rightarrow \{0, 1\}$ as

Display

$$f(v_i) = \begin{cases} 0 & i \quad odd \\ 1 & i \quad even \end{cases}$$

$$f(v_i{}') = \begin{cases} 1 & i \quad odd \\ 0 & i \quad even \end{cases}$$

$$f(v_i\, v_{i+1}) = 1 \qquad 1 \le i \le n-1$$

$$f(v_i\, v_{i+1}{}') = 0 \qquad 1 \le i \le n-1$$

$$f(v_i{}' v_{i+1}\,□) = 1 \qquad 1 \le i \le n-1$$

$$f(v_i□ v_{i+1}) = 0 \qquad 1 \le i \le n-1$$

end

f_0=no. of vertices and edges labelled 0;

f_1=no. of vertices and edges labelled 1;

display $|f_0-f_1|$

end

Theorem3.2.2: The Shadow graph $D_2(P_n)$ admits Total magic cordial labeling.

Proof: Let $D_2(P_n)$ be a graph with 2n vertices and 4(n-1) edges. The edge set and vertex set are explained in the above algorithm. To prove the Shadow graph $D_2(P_n)$ for every n, admits total sequential cordial labeling, we define a mapping f: $V \cup E \rightarrow \{0,1\}$ such that for all $(a,b) \in E$, $f(ab) = | f(a)-f(b)|$,as defined in step 3 of the above algorithm.

Let f_0 be the number of vertices and edges labelled zero and f_1 be the number of vertices and edges labelled one.

Case 1: When n is even

f_o=[n/2 +n/2 + n -1 +n -1]

f_1 =[n/2 +n/2 +n -1 +n -1]

then $|f_o - f_1| = |(3n-2)- (3n-2)| = 0.$

Case2: When n is odd

f_o=[((n+1)/2)+((n-1)/2) +n-1 +n-1]

f_1 = [((n-1)/2)+((n+1)/2) +n-1 +n-1]

then $|f_o - f_1| = |(3n-2) - (3n-2)| = 0.$

It shows that the number of vertices and edges labelled zero and number of vertices and edges labelled one differ at most by one in both cases. Hence the graph $D_2(P_n)$ admits Total sequential cordial labeling.

The total sequential cordial labeling of Shadow graph $D_2(K_{1,n})$ is given in the following algorithm.

Algorithm3.2.3

Step 1: Let V, V′ be the set of vertices
where $V = \{v_0, v_1, v_2, \ldots, v_n\}$ and
$V' = \{v_0', v_1', v_2', \ldots, v_n'\}$
Step 2: Let E_1, E_2, E_3 and E_4 be the set of edges
where $E_1 = \{v_0v_1, v_0v_2, \ldots, v_0v_n\}$,
$E_2 = \{v_0'v_1', v_0'v_2', \ldots, v_0'v_n'\}$,
$E_3 = \{v_0v_1', v_0v_2', \ldots, v_0v_n'\}$ and
$E_4 = \{v_0'v_1, v_0'v_2, \ldots, v_0'v_n\}$
Step 3: Define $f: V \cup E \rightarrow \{0, 1\}$ as
Display

$$f(v_0) = 0$$

$$f(v_i) = \begin{cases} 1 & i \quad odd \\ 0 & i \quad even \end{cases}$$

$$f(v_0') = 1$$

$$f(v_i') = \begin{cases} 0 & i \quad odd \\ 1 & i \quad even \end{cases}$$

$$f(v_0\, v_i) = \begin{cases} 1 & i \quad odd \\ 0 & i \quad even \end{cases}$$

$$f(v_0' v_i') = \begin{cases} 1 & i \quad odd \\ 0 & i \quad even \end{cases}$$

$$f(v_0 v_i') = \begin{cases} 0 & i \quad odd \\ 1 & i \quad even \end{cases}$$

$$f(v_0' v_i) = \begin{cases} 0 & i \quad odd \\ 1 & i \quad even \end{cases}$$

end
f_0=no. of vertices and edges labelled 0;
f_1=no. of vertices and edges labelled 1;
display $|f_0-f_1|$
end

Theorem3.2.4: The Shadow graph $D_2(K_{1,n})$ admits Total sequential cordial labeling.

Proof: Let $D_2(K_{1,n})$ be a graph with 2n+2 vertices and 4n edges. The edge set and vertex set are explained in the above algorithm. To prove the Shadow graph $D_2(K_{1,n})$ for every n, admits total sequential cordial labeling, we define a mapping $f: V \cup E \rightarrow \{0,1\}$ such that for all $(a,b) \in E$, $f(ab) = |f(a)-f(b)|$, as defined in step 3 of the above algorithm.

Let f_0 be the number of vertices and edges labeled zero and f_1 be the number of vertices and edges labelled one.

Case 1: When n is even
f_o=[1 +n/2 +n/2 + n/2 +n/2+ n/2 + n/2]
f_1 = [n/2 +1 +n/2 +n/2+n/2+n/2 +n/2]
then$|f_o - f_1| = |(3n+1)- (3n+1)| = 0$.
Case2: When n is odd
f_o=[1 +((n-1)/2)+((n+1)/2) +((n-1)/2) +((n-1)/2) +((n+1)/2) +((n+1)/2)]
f_1 = [((n+1)/2)+1 +((n-1)/2) +((n+1)/2) +((n+1)/2) +((n-1)/2) +((n-1)/2)]
then$|f_o - f_1| = |(3n+1) –(3n+1)| = 0$.

It shows that the number of vertices and edges labelled zero and number of vertices and edges labelled one differ at most by one in both cases. Hence the graph $D_2(K_{1,n})$ admits Total sequential cordial labeling.

V. CONCLUSION

We present an algorithm for getting total sequential cordial labeling for P_n^2, shadow graphs of path and star and we proved that the above graphs are total sequential cordial graphs.We would like to find other graphs that admits total sequential cordial labeling. Further the problem of cordial labeling for the same graph is under study.

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