Buyer - Vendor incentive inventory model with fixed lifetime product with fixed and linear back orders

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Abstract

The cooperation strategy followed by the buyer and vendor results is an overall effect on the saving percentage along with the impact of inventory costs such as holding cost, fixed and linear backorder cost. This paper deals with buyer - vendor incentive inventory model with fixed lifetime product with fixed and linear back order cost. A distinguishing feature of this model is that both fixed and linear backorder costs are included, where as previous work include without backorders. The model is developed and proved with a numerical example. The outcome of increasing holding cost, backorder cost for both buyer and vendor or individually is found.

Key words: Inventory, Quantity discount, Coordination, Fixed life time products, Fixed backorder cost, Linear back order cost

1. INTRODUCTION

The vendor - buyer situation is very unique in the business field and the production inventory decision is also very crucial in such situations. The buyer and vendor at times may face the situation of unfulfilled demand which is known as back order. The cost incurred by a business when it is unable to fill an order and must complete it later. A backorder cost can be discrete, as in the cost to replace a specific piece of inventory, or intangible, such as the effects of poor customer service. Backorder costs are usually computed and displayed on a per-unit basis. Backorder costs are important for companies to track, as the relationship between holding costs of inventory and backorder costs will determine whether a company should over- or under-produced. If the carrying cost of inventory is less than backorder costs, the company should over-produce and keep an inventory. The real and perceived costs of the inability to fulfill an order is the backorder cost. The costs can include negative customer relations, interest expenses, etc. The cost incurred by the vendor and buyer due to backorder is termed as backorder cost. The backorder cost is further classified as fixed and linear back order cost.

There will be a huge inventory loss in the case of health care industry, chemical industry, food and beverage industry due to perishability of either raw materials or finished product. The inventory cost that includes ordering cost, carrying cost, and shortage cost also increases due to difficulties in managing the perishable products and therefore there will be a customer dissatisfaction due to the cost and quality deterioration which spoils the image of the company. Liu and Shi (1999) classified perishability and deteriorating inventory models into two major categories namely decay models and finite life time models. The first model deals with the inventory that deteriorates and reduces in quantity continuously in proportion with time. The second model assumes a limited life time for each item. It is further classified into two subcategories namely fixed finite lifetime model and random finite lifetime model. Fixed life time items model deals with the perishable items while random life time model deals with probability distribution such as exponential and Erlang distribution. Fries (1975), Nandakumar and Morton (1993), Liu and Lian (1999), Lian and Liu (2001), developed the inventory models for fixed life time perishable problem. These researchers have mainly addressed single stage inventory system. Fujiwara et al (1997) studies the problem of ordering and issuing policies in controlling finite life time products, Kanchana and Anulark (2006) analyzed the effect of product perishability and retailers stock out policy of the inventory system. L.H. Chen and F.S. Kang (2010) analyzed the coordination inventory models for the vendor and buyer for trade credit items.

Supply chain management provides an important role for active cooperation and closed coordination therefore few mechanisms are applied to coordinate between parties. Some examples of these mechanisms are quantity discount, revenue sharing, sales rebate and trade credit. The quantity discount is a commonly used scheme among these mechanisms. Goyal and Gupta (1989) reviewed the literatures on the quantity discount model. Yongrui and Jianwen (2010) had researches on buyer-vendor inventory coordination with quantity discount for fixed life time products. We extend the model to consider discount and two types of back order costs to compare with Yongrui and Jianwen (2010). Cardenas-Barron (2011) and G.P. Sphicas (2006) developed an inventory model with fixed and linear backorder. W.K. Wong et al. (2009) analyzed supply chains with sales rebate contracts inventory model. Giannoccaro and Pontrandolfo (2004) developed supply chain coordination by revenue sharing contracts model.

Past researchers analyzed a single-vendor, single-buyer supply chain with fixed life time product without shortages. In this paper vendor, buyer supply chain with two types of back order costs is considered. The developed models analyze the benefit of coordinating supply chain by quantity discount strategy. If the coordination is not considered, given buyer's economic order quantity, the vendor's order size is an integer multiple of the buyer's that minimizes his own inventory cost. The vendor request the buyer to modify his current EOQ under the proposed coordination strategy and the vendor's order size is another integer multiple of the buyer's new order quantity. Now the vendor can benefit from lower setup, ordering and inventory holding cost. If the buyer accepts this offer, the vendor must compensate the buyer for his increased inventory cost and possibly provide an additional saving by offering the buyer a quantity discount, which depends on his order size. If we ignore backorders then we get the model by Yongrui and Jianwen (2010), which is considered a particular case in our model.

This paper deals with the holding cost, fixed and linear back order cost for both buyer and vendor. The impact of increase in holding cost of the vendor when fixed and linear cost remains constant is determined using the model. The saving percentage is affected due to the changes in the vendor's holding cost. The holding cost may increase for both the buyer and the vendor at the same time when fixed and linear remains same, which has an effect over the saving percentage. The effect is determined through the model.

Another situation dealt in this paper is the holding cost increase for the buyer alone where the fixed and linear back order cost is the same. The major aspect is the impact of increase in fixed and linear back order cost when the holding cost for buyer and vendor remains the same. Also the outcome on saving percentage in case increase in fixed back order cost where linear back order cost alone is also analyzed. The situation when holding cost increases either for buyer or vendor when back order also increase is considered for analysis. The result of saving percentage when holding cost for both buyer and vendor and fixed, linear back order cost increases is also determined.

The detailed description of this paper is as follows. In section 2, assumptions and notations, decentralized models with and without coordination models are given. Analytically easily understandable solutions are obtained in these models. It is proved that the quantity discount is the best strategy to achieve system optimization and win – win outcome. In section 3, a numerical example, algorithm and flow chart are given in detail to illustrate the models. Finally conclusion and summary are presented.

2. MODEL FORMULATION

In this section we analyzed decentralized models with and without coordination. In the coordination strategy quantity discount is offered by the vendor to the buyer.

2.1 ASSUMPTIONS AND NOTATIONS

2.1.1 Assumptions

(1) Demand rate is constant and known over the horizon planning

(2) Back orders are allowed and all back orders are satisfied.

(3) Two types of back order costs are considered. Linear back order cost (back order cost is applied to average backorders) and fixed cost (back order cost is applied to maximum back order level allowed).

(4) Lead time is zero.

(5) The model is for single product.

(6) All items ordered by the vendor arrive fresh and new. i.e., their age equals zero.

2.1.2 Notations

D : Annual demand of the buyer

L : Life time of product

k_1, k_2 : Vendor and buyer's setup costs per order, respectively

h_1, h_2 : Vendor and buyer's holding costs, respectively

p_1, p_2 : Delivered unit price paid by the vendor and the buyer respectively

B : Size of back orders in units.

Π : Back order cost per unit (fixed back order cost)

Π_1 : Back order cost per unit, per unit of time (linear back order cost)

m : Vendor's order multiple in the absence of any coordination

n : Vendor's order multiple under coordination

K : Buyer's order multiple under coordination. KQ_0 buyer's new order quantity

$d(K)$: Denotes the per unit dollar discount to the buyer if he orders KQ_0 every time

2.2 Model 1: EOQ model without coordination with fixed and linear backorders

Without coordination the buyer's total cost is formulated as follows

$$TC_b\ (Q, B) = \frac{Dk_2}{Q} + \frac{(Q-B)^2 h_2}{2Q} + \frac{\pi_1 B^2}{2Q} + \frac{\pi DB}{Q} \qquad (1)$$

$$\frac{\partial TC_b\ (Q,B)}{\partial Q} = Dk_2 \left(\frac{-1}{Q^2}\right) + \frac{(Q^2-B^2)h_2}{2Q^2} + \frac{\pi_1 B^2}{2}\left(\frac{-1}{Q^2}\right) + \pi DB \left(\frac{-1}{Q^2}\right) \text{ and}$$

$$\frac{\partial TC_b\ (Q,B)}{\partial B} = -\frac{(Q-B)h_2}{Q} + \frac{\pi_1 B}{Q} + \frac{\pi DB}{Q}$$

For optimality $\frac{\partial TC_b\ (Q,B)}{\partial Q} = 0$ and $\frac{\partial TC_b\ (Q,B)}{\partial B} = 0$

Now $Q_0^* = \sqrt{\frac{2Dk_2\ (h_2+\pi_1) - \pi^2\ D^2}{h_2 \pi_1}}$ and

$$B_0^* = \frac{h_2 Q^* - \pi D}{h_2 + \pi_1}$$

Total minimum cost of buyer

$$TC_b^0 = \frac{1}{h_2 + \pi_1}\left(\sqrt{2Dk_2\, h_2 \pi_1 (h_2 + \pi_1) - \pi^2\ D^2} + \pi D h_2\right)$$

Without any coordination, the buyer's order quantity is $Q_0 = \sqrt{\frac{2Dk_2\ (h_2+\pi_1) - \pi^2\ D^2}{h_2 \pi_1}}$ with the annual cost $TC_b = \frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\, h_2\pi_1 (h_2 + \pi_1) - \pi^2\ D^2} + \pi D h_2\right)$.

The vendor's order size is mQ_0, since he faced with a stream of demands at fixed intervals $t_0 = \sqrt{\frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}}$.

The average inventory for vendor's is

$[(m-1) Q_0 + (m-2) Q_0 + \ldots\ldots Q_0 + 0\, Q_0] / m$

$= (m-1) Q_0/ 2$

In the absence of coordination total annual cost for the vendor is

(1) Procurement cost = $\frac{Dk_1}{mQ_0}$ plus
(2) The annual average inventory holding cost $\frac{(m-1)(Q-B)^2 h_1}{2Q_0}$ plus
(3) Linear back order cost = $\frac{\pi_1 (m-1)B^2}{2Q_0}$ plus
(4) Fixed back order cost = $\frac{\pi DB}{Q_0}$

Thus

$$TC_v(m) = \frac{Dk_1}{mQ_0} + \frac{(m-1)(Q-B)^2 h_1}{2Q_0} + \frac{\pi_1 (m-1)B^2}{2Q_0} + \frac{\pi DB}{Q_0}$$

$$= \frac{1}{\sqrt{\frac{2Dk_2 (h_2+\pi_1)-\pi^2 D^2}{h_2\pi_1}}} \left[\frac{Dk_1}{m} + \frac{(m-1)\left(\frac{\pi_1 B+\pi D}{h_2}\right)^2 h_1}{2} + \frac{\pi_1 (m-1)B^2}{2} + \pi DB\right]$$

Without coordination vendor's problem can be developed as

min $TC_v(m)$

$$\text{w.r.t} \begin{cases} mt_0 \le L, \\ m \ge 1, \end{cases} \qquad (2)$$

where $mt_0 \le L$ which shows that items are not overdue before they are sold up by the buyer.

Theorem 1

If $L^2 \ge \frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}$, then

$$m^* = \min\left\{\left\lceil \sqrt{\frac{2Dk_1}{\left(\frac{\pi_1 B+\pi D}{h_2}\right)^2 h_1 + \pi_1 B^2} + \frac{1}{4}} - \frac{1}{2}\right\rceil, \left\lfloor \frac{L}{\sqrt{\frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}}} \right\rfloor\right\} \qquad (3)$$

where m^* be the optimum of (2) and $\lceil x \rceil$ is the least integer greater than or equal to x, $L^2 \ge \frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}$ is to ensure that $m^* \ge 1$.

Proof

Since $TC_v(m)$ is strictly convex in m we have

$$\frac{d^2 TCv(m)}{dm^2} = \frac{2Dk_1}{m^3} \sqrt{\frac{h_2 \pi_1}{2Dk_2 (h_2+\pi_1) - \pi^2 D^2}} > 0 .$$

Assume that m_1^* is an optimum of (2), then we have

$$m_1^* = \max\{\min\{m / TC_v(m) \le TC_v(m+1)\}, 1\}$$

$$= \max\{\min\{m / m(m+1) \ge \frac{2Dk_1}{(Q-B)^2 h_1 + \pi_1 B^2}\}, 1\}$$

$$= \left\lceil \sqrt{\frac{2Dk_1}{\left(\frac{\pi_1 B+\pi D}{h_2}\right)^2 h_1 + \pi_1 B^2} + \frac{1}{4}} - \frac{1}{2}\right\rceil \ge 1.$$

Applying $t_0 = \sqrt{\frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}}$ into the constraints in (1) of equation (2), then there exits the following inequality holds.

$$m\sqrt{\frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}} \le L$$

Consider $m_2^* = \frac{L}{\sqrt{\frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}}} \ge 1$ because $L^2 \ge \frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}$

Suppose $m_1^* \le m_2^*$ then $m^* = m_1^*$, otherwise $m^* = m_2^*$ where $TC_v(m)$ is a convex function.

Therefore if $L^2 \ge \frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}$, then $m^* =$

$$\min\left\{\left\lceil \sqrt{\frac{2Dk_1}{\left(\frac{\pi_1 B+\pi D}{h_2}\right)^2 h_1 + \pi_1 B^2} + \frac{1}{4}} - \frac{1}{2}\right\rceil, \left\lfloor \frac{L}{\sqrt{\frac{2k_2 (h_2+\pi_1)-\pi^2 D}{Dh_2\pi_1}}} \right\rfloor\right\}$$

Remark 1: Without any coordination the vendor places $\frac{D}{m^*\left(\sqrt{\frac{2Dk_2 (h_2+\pi_1)-\pi^2 D^2}{h_2\pi_1}}\right)}$ orders with a regular interval $\frac{m^*\left(\sqrt{\frac{2Dk_2 (h_2+\pi_1)-\pi^2 D^2}{h_2\pi_1}}\right)}{D}$.

The vendor order size is $m^*\left(\sqrt{\frac{2Dk_2\,(h_2+\pi_1)-\pi^2\,D^2}{h_2\pi_1}}\right)$ and the minimized total cost is $TC_v(m^*)$.

Model 2: EOQ model with coordination with fixed and linear backorders

On coordination strategy i.e., under quantity discount the vendor allows the buyer to change his current order size by KQ_0 with discount factor $d(K)$, K is the positive integer. The vendor's new order quantity is nKQ_0, where $n > 0$.

The total annual cost for the vendor is

(1) Procurement cost = $\frac{Dk_1}{nKQ_0}$ plus

(2) The annual average inventory holding cost = $\frac{(n-1)K(Q-B)^2h_1}{2Q_0}$ plus

(3) Linear back order cost = $\frac{\pi_1(n-1)KB^2}{2Q_0}$ plus

(4) Fixed back order cost = $\frac{\pi DB}{Q_0}$ plus

(5) The buyer's quantity discount which is equal to $Dd(K)p_2$

Therefore $TC_v(n) = \frac{Dk_1}{nKQ_0} + \frac{(n-1)K(Q-B)^2h_1}{2Q_0} + \frac{\pi_1(n-1)KB^2}{2Q_0} + \frac{\pi DB}{Q_0} + Dd(K)p_2$ (4)

Under coordination the problem can be developed as

$$\min TC_v(n) \quad (5)$$

w.r. t

$$\begin{cases} nKt_0 \le L, \\ \frac{Dk_2}{KQ_0} + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1 KB^2}{2Q_0} + \frac{\pi DB}{Q_0} \\ \quad -\frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\,h_2\pi_1(h_2+\pi_1) - \pi^2\,D^2} + \pi D h_2\right) \le p_2Dd(K) \\ n \ge 1, \end{cases}$$

where $nKt_0 \le L$ specifes that items are not overdue before they are sold up by the buyer. The next constraint specifies that the buyer's cost under coordination is less than the absence of any coordination.

Theorem 2

Let m^* be the optimum of (2) and n^* be the optimum of (5) then

$$TC_v(n^*) \le TC_v(m^*) \quad (6)$$

Proof

The quantity discount factor $p_2Dd(K)$ is a compensation given by the vendor to the buyer which is a part of the vendor's costs. If constraint of (5) is an equation then $TC_v(n)$ is minimum.

i.e.,

$$\frac{Dk_2}{KQ_0} + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1 KB^2}{2Q_0} + \frac{\pi DB}{Q_0} - \frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\,h_2\pi_1(h_2+\pi_1) - \pi^2\,D^2} + \pi D h_2\right)$$

$$= p_2Dd(K)$$

$$d(K) = \frac{\frac{Dk_2}{KQ_0} + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1 KB^2}{2Q_0} + \frac{\pi DB}{Q_0} - \frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\,h_2\pi_1(h_2+\pi_1) - \pi^2\,D^2} + \pi D h_2\right)}{p_2\,D} \quad (7)$$

Put $K = 1$ in (7), we have $d(1) = 0$

i.e., $d(1) =$

$$\frac{\frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\,h_2\pi_1(h_2+\pi_1) - \pi^2\,D^2} + \pi D h_2\right) - \frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\,h_2\pi_1(h_2+\pi_1) - \pi^2\,D^2} + \pi D h_2\right)}{p_2\,D}$$

$$= 0$$

If we have K = 1 in (5) we arrive (2) which is the special case of (5). Hence the inequality is true.

Remark 2: The above theorem proves that the buyer's cost with coordination is less than the without coordination. Therefore the vendor gets more benefited for the buyer's new order quantity.

Now we have find vendor and buyer optimal ordering quantity

Applying $d(K)$ into (4) we get

$$TC_v(n) = \frac{Dk_1}{nKQ_0} + \frac{(n-1)K(Q-B)^2h_1}{2Q_0} + \frac{\pi_1(n-1)KB^2}{2Q_0} + \frac{\pi DB}{Q_0}$$

$$+p_2\,D\left(\frac{\frac{Dk_2}{KQ_0} + \frac{K(Q-B)^2h_2}{2Q_0} + \frac{\pi_1 KB^2}{2Q_0} + \frac{\pi DB}{Q_0} - \frac{1}{h_2+\pi_1}\left(\sqrt{2Dk_2\,h_2\pi_1(h_2+\pi_1) - \pi^2\,D^2} + \pi D h_2\right)}{p_2\,D}\right)$$

(8)

Since (8) is a convex function, $d(K)$ is convex in K.

Consider the minimum of $\mathrm{TC_v(n)}$ is K^*. For optimality $\frac{\mathrm{dTC_v\ (n)}}{\mathrm{dK}} = 0$, we get

$$\mathrm{K}^*(\mathrm{n}) = \sqrt{\frac{2\mathrm{D}(\frac{\mathrm{k}_1}{\mathrm{n}}+\mathrm{k}_2)}{(\mathrm{Q}_0-\mathrm{B})^2[(\mathrm{n}-1)\mathrm{h}_1+\mathrm{h}_2]+\pi_1\mathrm{nB}^2}} \quad (9)$$

Now put

$\mathrm{K}^*(\mathrm{n}) = \sqrt{\frac{2\mathrm{D}(\frac{\mathrm{k}_1}{\mathrm{n}}+\mathrm{k}_2)}{(\mathrm{Q}_0-\mathrm{B})^2[(\mathrm{n}-1)\mathrm{h}_1+\mathrm{h}_2]+\pi_1\mathrm{nB}^2}}$ and $\mathrm{t}_0 = \sqrt{\frac{2\mathrm{k}_2\,(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}}{D\mathrm{h}_2\pi_1}}$ into the first constraint of (5) we get

$$2\mathrm{n}^2\left(\frac{\mathrm{k}_1}{\mathrm{n}}+\mathrm{k}_2\right)[2\mathrm{k}_2\,(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}] \leq \mathrm{L}^2\mathrm{h}_2\pi_1\left(\left(\frac{\pi_1 B+\pi\mathrm{D}}{\mathrm{h}_2}\right)^2[(\mathrm{n}-1)\mathrm{h}_1+\mathrm{h}_2]+\pi_1\mathrm{nB}^2\right)$$

Consider

$$\mathrm{f(n)} = [-4\mathrm{k}_2^2\mathrm{h}_2(\mathrm{h}_2+\pi_1)-2\mathrm{k}_2\mathrm{h}_2\pi^2\,\mathrm{D}]n^2 + [\mathrm{L}^2\pi_1\mathrm{h}_1\,(\pi_1 B+\pi\mathrm{D})^2+\mathrm{L}^2{\pi_1}^2{\mathrm{h}_2}^2\mathrm{B}^2-4\,\mathrm{k}_1\mathrm{k}_2\mathrm{h}_2(\mathrm{h}_2+\pi_1)+2\mathrm{k}_1\mathrm{h}_2\pi^2\,\mathrm{D}]\mathrm{n}+\mathrm{L}^2\pi_1(\mathrm{h}_2-\mathrm{h}_1)\,(\pi_1 B+\pi\mathrm{D})^2 \quad (10)$$

then $\mathrm{nKt}_0 \leq L$ is equivalent $\mathrm{f(n)} \geq 0$.

Apply $\mathrm{K}^*(\mathrm{n})$ and $\mathrm{t}_0 = \sqrt{\frac{2\mathrm{k}_2\,(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}}{D\mathrm{h}_2\pi_1}}$ into $\mathrm{TC_v(n)}$, we get

$$\mathrm{TC_v(n)} = \sqrt{\frac{\mathrm{h}_2\pi_1}{2\mathrm{k}_2\mathrm{D}(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}^2}\left[2\mathrm{D}\left(\frac{\mathrm{k}_1}{\mathrm{n}}+\mathrm{k}_2\right)[(\mathrm{Q}_0-\mathrm{B})^2[(\mathrm{n}-1)\mathrm{h}_1+\mathrm{h}_2]+\pi_1\mathrm{nB}^2]+4\pi^2\,\mathrm{D}^2\mathrm{B}^2\right]} - \left(\sqrt{\frac{2\mathrm{Dk}_2\,\mathrm{h}_2\pi_1(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}^2+\pi^2\,\mathrm{D}^2{\mathrm{h}_2}^2}{(\mathrm{h}_2+\pi_1)^2}}\right) \quad (11)$$

Therefore (5) is equivalent to

$$\min \mathrm{TC_v(n)}$$

$$\text{w.r.t} \begin{cases} f(n) \geq 0, \\ n \geq 1, \end{cases} \quad (12)$$

Since $\sqrt{x}$ is a strictly increasing function for $x \geq 0$, (12) is equivalent to

$$\min \widetilde{TC}_v(n) = \frac{\mathrm{h}_2\pi_1\mathrm{D}}{2\mathrm{k}_2\mathrm{D}(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}^2}\left[\left(\frac{\mathrm{k}_1}{\mathrm{n}}+\mathrm{k}_2\right)[(\mathrm{Q}_0-\mathrm{B})^2[(\mathrm{n}-1)\mathrm{h}_1+\mathrm{h}_2]+\pi_1\mathrm{nB}^2]+2\pi^2\,\mathrm{D}^2\mathrm{B}^2\right]$$

$$\text{w.r.t} \begin{cases} f(n) \geq 0, \\ n \geq 1, \end{cases} \quad (13)$$

We must discuss the properties of $\widetilde{TC}_v(n)$ and $f(n)$, to solve the above equation.

Since $\widetilde{TC}_v(n)$ is convex when $h_2 \geq h_1$, because

$$\widetilde{TC}_v{}''(n) = \frac{\mathrm{h}_2\pi_1\mathrm{D}}{2\mathrm{k}_2\mathrm{D}(\mathrm{h}_2+\pi_1)-\pi^2\,\mathrm{D}^2}\left[\frac{2\mathrm{k}_1(\mathrm{Q}_0-\mathrm{B})^2[\mathrm{h}_2-\mathrm{h}_1]}{n^3}\right] > 0$$

otherwise it is concave.

$f(n)$ is strictly concave because

$$f''(n) = -2[4\mathrm{k}_2^2\mathrm{h}_2(\mathrm{h}_2+\pi_1)-2\mathrm{k}_2\mathrm{h}_2\pi^2\,\mathrm{D}] < 0.$$

Proposition 1

Assume that the minimum of $\widetilde{TC}_v(n)$ is n_1^* for $\mathrm{n} \geq 1$, then we have

$$n_1^* = \begin{cases} \sqrt{\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4}}-\frac{1}{2}, & \frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)} \geq 2 \\ 1, & otherwise \end{cases} \quad (14)$$

Proof

$\widetilde{TC}_v(n_1^*) \leq \min\{\widetilde{TC}_v(n_1^*-1), \widetilde{TC}_v(n_1^*+1)\}$ is true if n_1^* is the minimum of $\widetilde{TC}_v(n)$, $\mathrm{n} \geq 1$.

Now $\widetilde{TC}_v(n_1^*) - \widetilde{TC}_v(n_1^*-1) = \left(\frac{\pi_1\mathrm{B}+\pi\mathrm{D}}{\mathrm{h}_2}\right)^2\left[\mathrm{k}_2\mathrm{h}_1-\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)}{n_1^*(n_1^*-1)}\right]+\pi_1\mathrm{k}_2\mathrm{B}^2 \leq 0$

we have,

$$\left(n_1^*-\frac{1}{2}\right)^2 \leq \frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4} \quad (15)$$

Similarly, by $\widetilde{TC}_v(n_1^*) - \widetilde{TC}_v(n_1^*+1) \leq 0$

we have

$$\left(n_1^*+\frac{1}{2}\right)^2 \geq \frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4} \quad (16)$$

Hence $n_1^* = \left[\sqrt{\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4}}-\frac{1}{2}\right]$

when $\sqrt{\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4}}-\frac{1}{2} \leq n_1^* \leq \sqrt{\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4}}+\frac{1}{2}$ (By (15) & (16)).

Otherwise $n_1^* = 1$ when $\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}+\frac{1}{4} < 0.$

Also note that

$n_1^* = 1,\ 0 < \frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1\,(\pi_1\mathrm{B}+\pi\mathrm{D})^2)} < 2.$

Therefore (14) is true.

Proposition 2

Let $n_{2(1)}^*$ and $n_{2(2)}^*$ be solution of (10), then

i) If $Y^2+4XZ < 0$ or $Y^2+4XZ \geq 0$ and $\mathrm{n}_{2(1)}^* < 1$, then $\mathrm{f(n)} < 0$ for $n \geq 1$.

ii) If $Y^2 + 4XZ \geq 0$ and $n^*_{2(1)} \geq 1$, then If $n^*_{2(2)} \geq 1$, $f(n) \geq 0$ for $\lceil n^*_{2(2)} \rceil \leq n \leq \lceil n^*_{2(1)} \rceil$; If $n^*_{2(2)} < 1$ *and* $n^*_{2(1)} \geq 1$, $f(n) \geq 0$ for $1 \leq n \leq \lceil n^*_{2(1)} \rceil$;

where $X = -4k_2^2 h_2(h_2 + \pi_1) - 2k_2 h_2 \pi^2 D$, $Y = L^2 \pi_1 h_1 (\pi_1 B + \pi D)^2 + L^2 {\pi_1}^2 {h_2}^2 B^2 - 4 k_1 k_2 h_2 (h_2 + \pi_1) + 2k_1 h_2 \pi^2 D$ and

$Z = L^2 \pi_1 (h_2 - h_1)(\pi_1 B + \pi D)^2$

Proof

Now solve the quadratic equation $f(n) = 0$,

we get

$$n^*_{2(1)} = \frac{Y + \sqrt{Y^2 + 4XZ}}{2X} \text{ and } n^*_{2(2)} = \frac{Y - \sqrt{Y^2 + 4XZ}}{2X}$$

The following conclusions are true because f(n) is an quadratic equation.

1) $f(n) < 0$ where $Y^2 + 4XZ < 0$ for every n.
2) $n^*_{2(1)}$ and $n^*_{2(2)}$ are real solutions of $f(n) = 0$ where $Y^2 + 4XZ \geq 0$.In view of $n \geq 1$,
   i) $f(n) < 0$ where $n^*_{2(1)} < 1$, for $n \geq 1$;
   ii) $f(n) \geq 0$ where $n^*_{2(2)} \geq 1$, for $\lceil n^*_{2(2)} \rceil \leq n \leq \lceil n^*_{2(1)} \rceil$;
   iii) $f(n) \geq 0$ where $n^*_{2(2)} < 1$ and $n^*_{2(1)} \geq 1$, for $1 \leq n \leq \lceil n^*_{2(1)} \rceil$.

Remark 3: If $\lceil n^*_{2(2)} \rceil \leq n \leq \lceil n^*_{2(1)} \rceil$ or $1 \leq n \leq \lceil n^*_{2(1)} \rceil$ then the conclusion (ii) of proposition 2 and $nKt_0 \leq L$ is true. The problem is meaningless if (i) of proposition 2 is true and the first constraint of (5) does not true for any $n \geq 1$.

Theorem 3

i) $n^* = n_1^*$ if $1 \leq n_1^* \leq \lceil n^*_{2(1)} \rceil$
ii) ii) $n^* = \lceil n^*_{2(1)} \rceil$ if $n_1^* > \lceil n^*_{2(1)} \rceil$ where $h_2 \geq h_1$ and $n^*_{2(2)} \geq 1$.

Proof

Since n_1^* is the minimum of $\widetilde{TC}_v(n)$ for $n \geq 1$ then $\widetilde{TC}_v(n)$ is a convex function.

Hence if $n^* = n_1^*, 1 \leq n_1^* \leq \lceil n^*_{2(1)} \rceil$ else $n^* = \lceil n^*_{2(1)} \rceil, n_1^* > \lceil n^*_{2(1)} \rceil$.

Here $\widetilde{TC}_v(n)$ is decreasing on the interval $n_1^* > \lceil n^*_{2(1)} \rceil$ *so* $n^* = \lceil n^*_{2(1)} \rceil$.

Remark 4: $\widetilde{TC}_v(n)$ is strictly concave if the vendor's unit holding cost is higher than the buyer's. This is not common in practice so we will not give further discussion about this.

Theorem 4

If $h_2 \geq h_1$ then $K^*(n^*) > 1$.

Proof

$$K^*(n) = \sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{(Q_0 - B)^2[(n-1)h_1 + h_2] + \pi_1 n B^2}}$$

$$= \sqrt{\frac{2D\left(\frac{k_1}{n} + k_2\right)}{\left(\frac{\pi_1 B + \pi D}{h_2}\right)^2[(n-1)h_1 + h_2] + \pi_1 n B^2}}$$

(I) If $\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)} \geq 2$ then $n^* = n_1^*$.

i.e., $n^* = n_1^* = \left\lceil \sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)} + \frac{1}{4}} - \frac{1}{2} \right\rceil$.

$K^*(n^*)$ is a decreasing function of n if $\left\lceil \sqrt{x + \frac{1}{4}} - \frac{1}{2} \right\rceil \leq \sqrt{x} + 1$ is true for $x \geq 0$.

To prove $K^*\left[\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}} + 1\right] > 1$

i.e.,

$$\sqrt{\frac{2D\left(\frac{k_1}{\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}} + 1\right)} + k_2\right)}{\left(\frac{\pi_1 B + \pi D}{h_2}\right)^2\left[\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}}\right)h_1 + h_2\right] + \pi_1\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}} + 1\right)B^2}}$$

$=$

$$\frac{2D\left(k_1 + k_2\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}} + 1\right)\right)}{\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}} + 1\right)\left\{\left(\frac{\pi_1 B + \pi D}{h_2}\right)^2\left[\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}}\right)h_1 + h_2\right] + \pi_1\left(\sqrt{\frac{k_1(h_2 - h_1)(\pi_1 B + \pi D)^2}{k_2(\pi_1 B^2 h_2^2 + h_1(\pi_1 B + \pi D)^2)}} + 1\right)B^2\right.}$$

> 1

$$2D\mathrm{k}_1 + 2\mathrm{Dk}_2\sqrt{\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}} + 2D\mathrm{k}_2 > \left(\frac{\pi_1 B+\pi\mathrm{D}}{\mathrm{h}_2}\right)^2\left(\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}\right)\mathrm{h}_1 + \left(\frac{\pi_1 B+\pi\mathrm{D}}{\mathrm{h}_2}\right)^2\sqrt{\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}}\,\mathrm{h}_2 + \pi_1 B^2\left(\frac{\mathrm{k}_1(\mathrm{h}_2-\mathrm{h}_1)(\pi_1\mathrm{B}+\pi\mathrm{D})^2}{\mathrm{k}_2(\pi_1\mathrm{B}^2\mathrm{h}_2^2+\mathrm{h}_1(\pi_1\mathrm{B}+\pi\mathrm{D})^2)}\right) \quad (17)$$

Hence (17) holds if $k_1, k_2, h_1, h_2\, B, D, \pi_1, \pi$ are all positive and $h_2 \geq h_1$.

(II) If $n^* = n_1^* = 1$then

$$K^*(1) = \sqrt{\frac{2Dh_2(k_1+k_2)}{(\pi_1\mathrm{B}+\pi\mathrm{D})^2+\pi_1\mathrm{B}^2 h_2}}$$

Hence $K^*(1) > 1$ if $k_1, k_2, h_2\, B, D, \pi_1, \pi$ are all positive.

(III) If $n^* = \lceil n_{2(1)}^* \rceil, n_1^* > \lceil n_{2(1)}^* \rceil$ then $K^*(\lceil n_{2(1)}^* \rceil) \geq \mathrm{K}^*(n_1^*) > 1$.

From (I) to (III), $K^*(n) > 1$ if $h_2 \geq h_1$.

Remark 5: Theorem (4) specifies that the buyer's order size is greater to compare with cooperation against the non-cooperation if $h_2 \geq h_1$.

3. NUMERICAL EXAMPLES

In this section, numerical examples are presented to illustrate the performance of the quantity discount strategy proposed in previous section. The sensitivity analysis of cost savings on parameters has been given.

The buyer's saving in percentage

$\mathrm{SP_b} = 100 \propto (\mathrm{TC_v}(m^*) - \mathrm{TC_v}(n^*))/\mathrm{TC_b}(m^*)$.

The vendor's saving in percentage

$\mathrm{SP_{v1}} = 100(1-\propto)(\mathrm{TC_v}(m^*) - \mathrm{TC_v}(n^*))/\mathrm{TC_v}(m^*)$.

The vendor's saving in percentage if he does not share the saving with the buyer

$\mathrm{SP_{v2}} = 100(\mathrm{TC_v}(m^*) - \mathrm{TC_v}(n^*))/\mathrm{TC_v}(m^*)$.

Examples:

1. Given D = 10,000 units per year, p_2 = 30\$ per unit, α = 0.5, L = 0.25 year, k_1 = 300\$ per order, k_2 = 100\$, π = 0.01, π_1 = 1.0. The different values of h_1, h_2 and computational results are as specified in Table 1.

2. Given D = 10,000 units per year, p_2 = 30\$ per unit, α = 0.5, L = 0.25 year, k_1 = 300\$ per order, k_2 = 100\$, h_1 = 5, h_2 = 10. The different values of π, π_1 and computational results are as specified in Table 2.

3. Given D = 10,000 units per year, p_2 = 30\$ per unit, α = 0.5, L = 0.25 year, k_1 = 300\$ per order, k_2 = 100\$, h_2 = 10. The different values of h_1, π, π_1 and computational results are as specified in Table 3.

4. Given D = 10,000 units per year, p_2 = 30\$ per unit, α = 0.5, L = 0.25 year, k_1 = 300\$ per order, k_2 = 100\$, h_1 = 5. The different values of h_2, π, π_1 and computational results are as specified in Table 4

5. Given D = 10,000 units per year, p_2 = 30\$ per unit, α = 0.5, L = 0.25 year, k_1 = 300\$ per order, k_2 = 100\$. The different values of h_1, h_2, π, π_1 and computational results are as specified in Table 5

Tables: Given in bottom of manuscript

The crux of the model is

1. Savings percentage remains constant when holding cost for vendor increases and back order costs are constant.
2. Savings percentage increases when holding cost of buyer increases when back order costs are constant.
3. Savings percentage increases when holding cost for both buyer and vendor increases and back order costs remains constant.
4. Savings percentage decreases when linear back order cost increases and holding cost and fixed backorder cost remains same.
5. Savings percentage decreases when fixed back order increases and holding cost and linear backorder cost remains same.

6. Savings percentage decreases when back order costs are increased and holding cost remains constant.

7. Savings percentage decreases when backorder cost and holding cost of either vendor or buyer increases.

8. Savings percentage decreases when both holding cost and backorder cost increases.

The computational result indicates that the system can save more cost under coordination.

3.1 ALGORITHM AND FLOWCHART

3.1.1: Algorithm

Step 1: Initialize the values

Step 2: Read the values

Step 3: Find Q_0, B_0

Step 4: If $\frac{k_1(h_2-h_1)(\pi_1 B+\pi D)^2}{k_2(\pi_1 B^2 h_2^2+h_1(\pi_1 B+\pi D)^2)} \geq 2$ then go to step 5 otherwise go to step 6

Step 5:

i) Find n^* by using the equation first constrain of (14)

ii) Find $K^*(n)$ by using the equation (9)

iii) Find $TC_V(n^*)$ by using the equation (8)

iv) Find optimum m^* by using the equation (3)

v) Find $TC_V(m^*)$ by using (2)

vi) Find TC_b

vii) Calculate SP_b, SP_{v1}, SP_{v2}

Step 6: i) Initialize n = 1 then

ii) Go to step 5(ii) – (vii)

Step 7: End

Figures: Given in bottom of manuscript

4. CONCLUSION

In this paper, we have developed inventory model in which quantity discount coordination strategy with linear and fixed backorder cost for a buyer - vendor supply chain of fixed life time product. Analytically proved and optimized decisions are arrived using the model. The cooperation strategy always increases the saving percentage of both vendor and the buyer. The various situations of changes especially increase in holding cost, backorder cost both linear as well as fixed is dealt in the paper. The model brings into light that the saving percentage increases when holding cost increase for both buyer and vendor or buyer alone and backorder cost remains the same. This situation is beneficial to both buyer and vendor and increase the profit for them. There is no change in saving percentage when vendor alone increases holding cost and backorder cost remains the same. The increase in backorder cost in any form either for the vendor or buyer or both leads to a decrease in saving percentage. Thus it could be concluded with numerical proof that an increase in backorder cost will reduce the benefit for both buyer and vendor with coordination. Another useful finding of the study is that the impact if an increase in backorder cost is much higher than the impact of increasing in holding cost. Thus backorder cost both fixed and linear costs are equally important from the cost and benefit point of view for the buyer and vendor. It has been proved that the buyer's order size is higher with cooperation than the non cooperation. The vendor gives order size dependent on discount to the buyer to compensate his increased inventory cost. We prove that the decentralized quantity discount strategy can achieve system optimization and win-win outcome. As a result both the vendor and the buyer benefit in the long run. Numerical example is presented to illustrate the model. Even though we consider the backorder cost, the system cost is reduced in comparison with the model by Yongrui and Jianwen (2010). Hence we obtained more savings for both the vendor and the buyer in our models with and without coordination

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TABLES

Table 1: Computational results for different values of h_1 and h_2

h_1	h_2	π	π_1	d(K)	Q	B	SP_b	SP_{v1}	SP_{v2}
2	5	0.01	1.0	0.0019	1548.5	1273.8	14.8910	10.1327	20.2654
3	5	0.01	1.0	0.0019	1548.5	1273.8	14.8910	10.1327	20.2654
4	5	0.01	1.0	0.0019	1548.5	1273.8	14.8910	10.1327	20.2654
5	5	0.01	1.0	0.0019	1548.5	1273.8	14.8910	10.1327	20.2654
3	6	0.01	1.0	0.0018	1527.0	1294.6	16.2201	11.0401	22.0803
4	7	0.01	1.0	0.0017	1511.4	1310.0	17.1861	11.7001	23.4001
5	8	0.01	1.0	0.0016	1499.6	1321.9	17.9206	12.2020	24.4039
6	9	0.01	1.0	0.0016	1490.3	1331.3	18.4982	12.5967	25.1935
7	10	0.01	1.0	0.0016	1482.9	1339.0	18.9643	12.9155	25.8309
5	5	0.01	1.0	0.0019	1548.5	1273.8	14.8910	10.1327	20.2654
5	6	0.01	1.0	0.0018	1527.0	1294.6	16.2201	11.0401	22.0803
5	7	0.01	1.0	0.0017	1511.4	1310.0	17.1861	11.7001	23.4001
5	8	0.01	1.0	0.0016	1499.6	1321.9	17.9206	12.2020	24.4039
5	9	0.01	1.0	0.0016	1490.3	1331.3	18.4982	12.5967	25.1935
5	10	0.01	1.0	0.0016	1482.9	1339.0	18.9643	12.9155	25.8309
5	15	0.01	1.0	0.0014	1460.4	1362.8	20.3863	13.8887	27.7762
5	20	0.01	1.0	0.0014	1449.0	1375.2	21.1114	14.3842	28.7685

Table 2: Computational results for different values of π and π_1

h_1	h_2	π	π_1	d(K)	Q	B	SP_b	SP_{v1}	SP_{v2}
5	10	0.01	1.0	0.0016	1482.9	1339.0	18.9643	12.9155	25.8309
5	10	0.01	1.1	0.0017	1420.3	1270.5	18.5977	12.6528	25.3056
5	10	0.01	1.2	0.0018	1366.0	1210.7	18.2247	12.3878	24.7757

5	10	0.01	1.3	0.0019	1318.2	1157.7	17.8470	12.1214	24.2428
5	10	0.01	1.4	0.0020	1275.9	1110.4	17.4659	11.8540	23.7081
5	10	0.01	1.5	0.0021	1238.0	1067.8	17.0824	11.5862	23.1724
5	10	0.01	1.6	0.0022	1203.9	1029.2	16.6971	11.3182	22.6364
5	10	0.02	1.0	0.0016	1481.9	1329.0	17.8805	12.4144	24.8288
5	10	0.03	1.0	0.0015	1480.2	1318.4	16.9452	11.9731	23.9462
5	10	0.04	1.0	0.0015	1477.8	1307.1	16.1348	11.5847	23.1694
5	10	0.05	1.0	0.0015	1474.8	1295.3	15.4306	11.2434	22.4869
5	10	0.02	1.1	0.0017	1419.3	1260.7	17.5855	12.1870	24.3740
5	10	0.03	1.2	0.0018	1363.5	1190.6	16.4538	11.5707	23.1415
5	10	0.04	1.3	0.0019	1313.8	1127.3	15.5082	11.0436	22.0872
5	10	0.05	1.4	0.0020	1269.1	1069.4	14.7079	10.5891	21.1783
5	10	0.06	1.5	0.0021	1228.5	1016.1	14.0240	10.1954	20.3908

Table 3: Computational results for different values of h_1, π and π_1

h_1	h_2	π	π_1	d(K)	Q	B	SP_b	SP_{v1}	SP_{v2}
5	10	0.01	1.0	0.0016	1482.9	1339.0	18.9643	12.9155	25.8309
6	10	0.02	1.1	0.0017	1419.3	1260.7	17.5855	12.1870	24.3740
7	10	0.03	1.2	0.0018	1363.5	1190.6	16.4538	11.5707	23.1415
8	10	0.04	1.3	0.0019	1313.8	1127.3	15.5082	11.0436	22.0872
9	10	0.05	1.4	0.0020	1269.1	1069.4	14.7079	10.5891	21.1783
10	10	0.06	1.5	0.0020	1228.5	1069	14.0240	10.1954	20.3908

Table 4: Computational results for different values of h_2, π and π_1

h_1	h_2	π	π_1	d(K)	Q	B	SP_b	SP_{v1}	SP_{v2}
5	5	0.01	1.0	0.0019	1585	1273.8	14.8910	10.1327	20.2654

5	6	0.02	1.1	0.0019	1464	1209.6	14.7990	10.2450	20.4901
5	7	0.03	1.2	0.0019	1393	1152.9	14.6258	10.2757	20.5513
5	8	0.04	1.3	0.0020	1331	1102.4	14.4254	10.2657	20.5314
5	9	0.05	1.4	0.0020	1277	1057.1	14.2213	10.2354	20.4709
5	10	0.06	1.5	0.0021	1229	1016.1	14.0240	10.1954	20.3908

Table 5: Computational results for different values of h_1, h_2, $\boldsymbol{\pi}$ and $\boldsymbol{\pi}_1$

h_1	h_2	π	π_1	d(K)	Q	B	SP_b	SP_{v1}	SP_{v2}
5	10	0.01	1.0	0.0016	1482.9	1339.0	18.9643	12.9155	25.8309
6	11	0.02	1.1	0.0016	1413.0	1268.1	17.9752	12.4589	24.9178
7	12	0.03	1.2	0.0017	1351.7	1206.1	17.1784	12.0848	24.1696
8	13	0.04	1.3	0.0018	1297.2	1151.3	16.5226	11.7731	23.5462
9	14	0.05	1.4	0.0018	1248.5	1102.5	15.9732	11.5099	23.0197
10	15	0.06	1.5	0.0019	1204.4	1058.6	15.5064	11.2851	22.5702

FIGURES

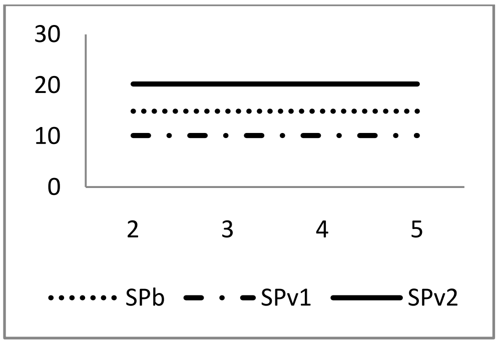

Figure 1(a): Effect of changes when holding cost for vendor increase (Table 1)

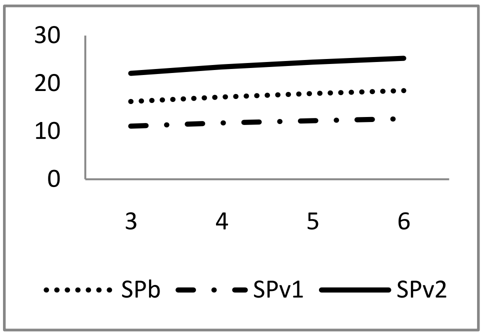

Figure 1(b): Effect of changes when holding cost for vendor and buyer increases (Table 1)

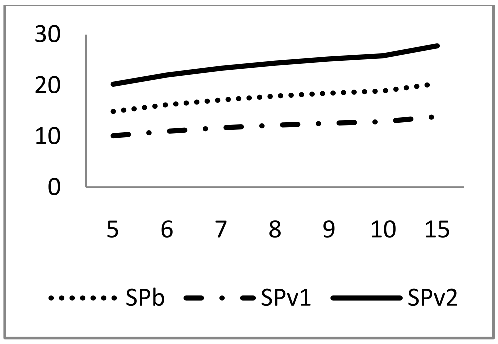

Figure 1(c): Effect of changes when holding cost for buyer increase (Table 1)

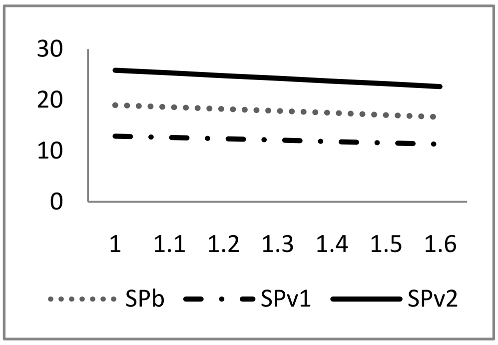

Figure 2(a): Effect of changes when linear backorder cost increase (Table 2)

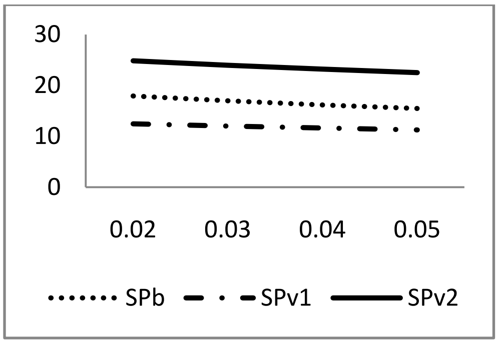

Figure 2(b): Effect of changes when fixed backorder cost increases (Table 2)

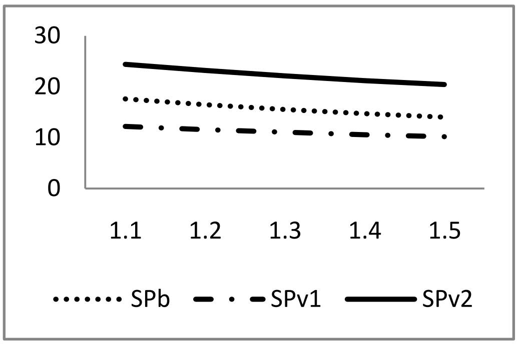

Figure 2(c): Effect of changes when both fixed and linear backorder cost increase (Table 2)