

# Design of Adaptive Controller for Vibration Control of Piezo Actuated Cantilever Beam

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## **Abstract—**

The reductions of noise, caused by structural vibrations and the diminishing of devastating vibrations have been of great interest. The design of controllers for smart actuated structures is a challenging problem due to the presence of non-linearities in the structural system and actuators, the limited availability of control force, and the non-availability of accurate mathematical models. The application of adaptive control algorithms for vibration suppression of smart structures is investigated in this paper. The adaptive controller adapts to the parameter variations of the structural system by updating the controller gains. Self tuning regulator is used to force the vibration level with-in the minimum acceptable limit. In this paper, adaptive control algorithms are investigated for designing active controllers for smart structures. Simulation studies presented in this thesis indicate the superior performance of the adaptive controller for vibration suppression.

**Keywords:** Adaptive controller, Self tuning regulator (STR), Smart structures

## **I. INTRODUCTION**

The field of smart structures has been an emerging area of research for the last few decades. Smart structures can be defined as structures that are capable of sensing and actuating in a controlled manner in response to a stimulus. The development of this field is supported by the development in the field of materials science and in the field of control. In materials science, new smart materials are developed that allow them to be used for sensing and actuation in an efficient and controlled manner. These smart materials are to be integrated with the structures so they can be employed as actuators and sensors effectively. It is also clear that the field of smart structures also involves the design and implementation of the control systems on the structures. A well designed and easily implementable controller for smart structures is thus desirable.

Governed by the need of lightweight solutions for the aerospace industry, active vibration control techniques have experienced rapid developments in the last thirty years. In this paper, we consider the case of vibration of smart structures. The stimulus to a structure may originate from external disturbances or excitations that cause structural vibrations. A smart structure would be able to sense the vibration and generate a controlled actuation to it so that the vibration can be minimized. For

vibration control purposes, a number of smart materials can be used as actuators such as piezoelectric, shape memory, electrostrictive and magnetostrictive materials. Here, we concentrate on using piezoelectric materials because they have good broadband sensing and actuation properties. The ability of the piezoelectric materials to exchange electrical and mechanical energy opens up the possibility of employing them both as actuators and sensors. If the piezoelectric materials are bonded properly to a structure, structural deformations can be induced by applying a voltage to the materials, employing them as actuators. On the other hand, they can be employed as sensors since deformations of a structure would cause the deformed piezoelectric materials to produce an electric charge. The extent of structural deformation can be observed by measuring the electrical voltage the materials produce. Unfortunately, the piezoelectric effect in natural crystals is rather weak so they cannot be used effectively as actuators or sensors. So artificially made piezoelectric material (PZT-5H) is considered for vibration control in this paper.

## **II. THE EXPERIMENTAL SCHEME:**

A flexible aluminum beam with a fixed clamped end as shown in fig 1 is considered here. This Experimental setup is taken from the reference journal [1].Two

piezoceramic patches are surface bonded at a distance of 5mm from the fixed end of the beam.

Table 1: Dimensions and properties of the Aluminum beam and Piezoceramic patches

Aluminium beam		piezoceramic sensor / actuator	
Length (m)	0.480	Length (m)	0.0765
Width(m)	0.01322	Width(m)	0.0127
Thickness(m)	0.00124	Thickness(m)	0.005
Young's modulus (GPa)	71	Young's modulus (GPa)	47.62
Density (Kg/m <sup>2</sup> )	2700	Density (Kg/m <sup>2</sup> )	7500
First natural frequency	6.26	piezorlectric strain constant (mV <sup>-1</sup> )	-247*10 <sup>-12</sup>
		piezorlectric stress constant (VmV <sup>-1</sup> )	-9*10 <sup>-3</sup>

The patch bonded on the bottom surface acts as a sensor and the one on the top surface acts as an actuator. An excitation input is applied to the structure through another piezoceramic patch which is bonded on the top surface at a distance of 370 mm from the fixed end. The dimensions and properties of the beam and piezoceramic patches are given in table 1.

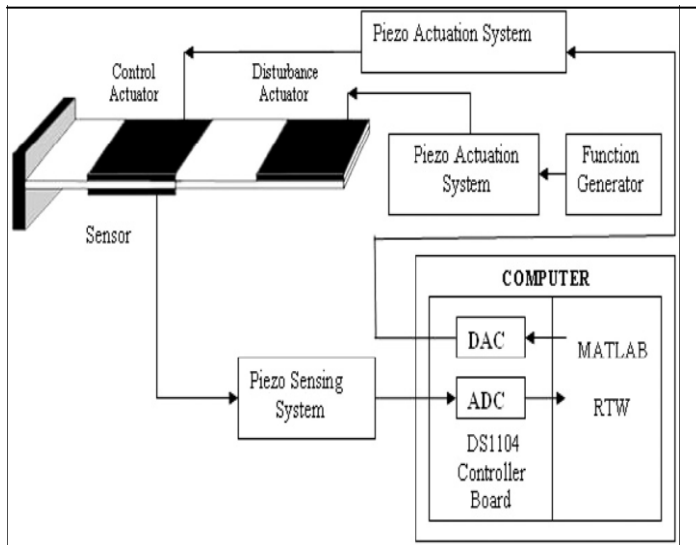


Fig. 1 Experimental setup

### III. IDENTIFICATION OF THE STRUCTURE DYNAMICS

The linear dynamic model of smart cantilever beam considered in this paper is referred from [ref] in which the model was identified using RLS parameter estimation method based on Auto-Regressive (ARX) model.

The continuous state space model derived from the identified second order ARX model parameters is given as

$$\dot{x} = Ax + bu + er; \quad y = c^T x, \quad --(1)$$

$$A = \begin{bmatrix} -83.0583 & 218.2890 \\ -204.9014 & 76.7292 \end{bmatrix}$$

$$B = \begin{bmatrix} -1.4349 \\ -1.708 \end{bmatrix}$$

$$e = \begin{bmatrix} -0.2359 \\ -0.0477 \end{bmatrix}$$

$$C^T = [(1) \quad 0]$$

### IV. PID CONTROLLER DESIGN:

In this section a design procedure is given for PID controller for vibration control. We assume initially that the PID controllers has a transfer function given by

$$D(w) = K_p + \frac{K_i}{w} + K_d w \quad --(2)$$

The design problem is to choose  $D(w)$  such that

$$D(j\omega_1)G(j\omega_1) = 1 \angle 180 + \phi_m \quad --(3)$$

at a chosen frequency  $\omega_{w1}$ .

$$K_p + j \left( K_D \omega_{w1} - \frac{K_I}{\omega_{w1}} \right) = |D(j\omega_{w1})|(\cos\theta + j\sin\theta) \quad --(4)$$

Therefore

$$K_p = |D(j\omega_{w1})|\cos\theta = \frac{\cos\theta}{|G(j\omega_{w1})|} \quad --(5)$$

$$K_{D\omega_{w1}} - \frac{K_I}{\omega_{w1}} = \frac{\sin\theta}{|G(j\omega_{w1})|} \quad --(6)$$

### V. ADAPTIVE CONTROLLER

Adaptive control has been under intense investigation in the last fifty years. Early developments were driven from the need of autopilots for high-

performance airplanes [Astrom, 1989]. Researchers found that conventional static-gain feedback controllers could not work well over the wide range of speeds and altitudes. In the 1960s, adaptive control techniques experienced a revolution with the introduction of the state space and stability theory. Today, many commercial solutions are on the market implementing adaptive techniques with digital microcontrollers. These concepts suggest that an adaptive system should include parameter adjustments to counter variations in the dynamics of the process or of the plant. In most cases, the achieved behavior is compared to the desired behavior to obtain a measurement of the performance. Therefore, adaptive control offers a solution for significant variations in the process dynamics and for variations in the disturbances. Due to the parameter adjustments within the controller, adaptive systems are highly non-linear. This can make the analysis for global stability of the system very complicated. Adaptive control systems have two loops A normal feedback with process and controller and a parameter adjustment loop (slower dynamics).

#### A. Adaptive Schemes:

Several adaptive schemes have been developed since research on adaptive systems began in the 1950s. Those schemes include gain scheduling, model reference adaptive control (MRAC), self-tuning regulators (STR). In this paper STR technique is used for suppress.

#### B. Self-Tuning Regulator (STR):

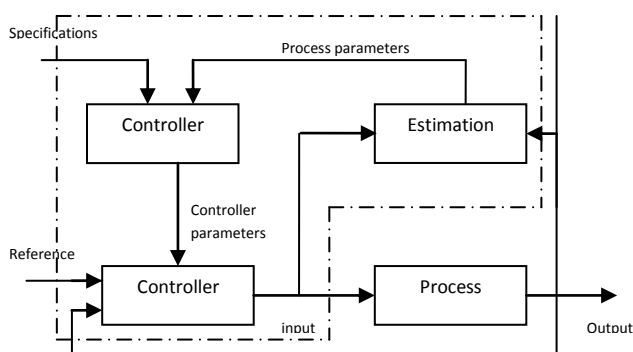


Fig. 2 Block diagram of Self Tuning Regulator (STR)

All schemes so far are called direct methods because the adjustments rules tell directly show the regulator parameters should be updated. The self-tuning regulator is a different scheme where process parameters

are up dated and the regulator parameters are obtained from the solution of a design problem. A block diagram of such a system is shown in Fig 2. The adaptive regulator can be thought of as composed of two loops. The inner loop consists of the process and an ordinary linear feedback regulator. The parameters of the regulator are adjusted by the outer loop, which is composed of a recursive parameter estimator and a design calculation. The block labeled "design" in Fig.2 represents an on-line solution to a design problem for a system with known parameters. This is called the underlying design problem. Such a problem can be associated with most adaptive control schemes. However, the problem is often given indirectly. The self-tuner also contains a recursive parameter estimator. Many different estimation schemes have been used, for example stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering, and the maximum likelihood method. The self-tuner shown in Fig.2 is called an explicit STR or an STR based on estimation of an explicit process model. In terms of Fig.2, the block labeled design calculations disappears and the regulator parameters are updated directly. It is sometimes possible to re parameterize the process so that it can be expressed in terms of the regulator parameters. This gives as significant simplification of the algorithm because the design calculations are eliminated.

#### C. Controller Design

##### 1 Pole Placement Design

The idea of controller design is, e.g. like MRAS, to determine a controller that gives the desired closed loop poles. The assumption for controller design is that the process is described by a Single-Input, Single Output system (SISO), as given in formula

$$A(q)y(t) = B(q)(u(t) + v(t)) \quad --(7)$$

A general linear controller for SISO systems is given in formula

$$Ru(t) = Tu_c(t) - Sy(t) \quad --(8)$$

where R, T and S are polynomials.

Fig.3. shows a SISO-system controlled by the linear controller of formula

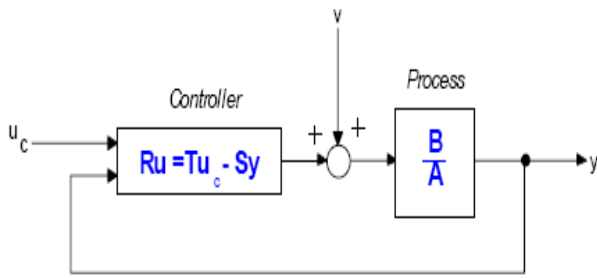


Fig.3 Process Model block

$$R = \frac{b_1}{b_0} + \frac{(b_1^2 - a_{m1}b_0b_1 + a_{m2}b_0^2)(-b_1 + a_0b_0)}{b_0(b_1^2 - a_1b_0b_1 + a_2b_0^2)}$$

$$s_0 = \frac{b_1(a_0a_{m1} - a_2 - a_{m1}a_1 + a_1^2 + a_{m2} - a_1a_0)}{b_1^2 - a_1b_0b_1 + a_2b_0^2} + \frac{b_0(a_{m1}a_2 - a_1a_2 - a_0a_{m2} + a_0a_2)}{b_1^2 - a_1b_0b_1 + a_2b_0^2}$$

$$s_1 = \frac{b_1(a_1a_2 - a_{m1}a_2 + a_0a_{m2} - a_0a_2)}{b_1^2 - a_1b_0b_1 + a_2b_0^2} + \frac{b_0(a_2a_{m2} - a_2^2 - a_0a_{m2}a_1 + a_0a_2a_{m1})}{b_1^2 - a_1b_0b_1 + a_2b_0^2}$$

$$T = \beta \left( \frac{-b_1}{b_0} + a_0 \right), \quad \beta = \frac{b_{m0}}{b_0}$$

After combining formulae and eliminating parameter  $u$ , the closed loop equations becomes as given in formula

$$y(t) = \frac{BT}{AR + BS} u_c(t) + \frac{BR}{AR + BS} v(t)$$

$$u(t) = \frac{AT}{AR + BS} u_c(t) + \frac{BS}{AR + BS} v(t)$$

--(9)

Consequently, the closed-loop characteristic polynomial is:

$$AR + BS = A_c \quad \text{--(10)}$$

This equation is called the Diophantine equation. The polynomials  $R$  and  $S$  can be solved from this Diophantine equation. The Diophantine equation only determines the polynomials  $R$  and  $S$ , therefore other conditions have to be introduced to find polynomial  $T$ . For this, a reference model that describes the response from the command signal  $u_c$  is needed. This is called the 'Model-following condition'. The Model-following condition will be described in the next subparagraph to determine the control polynomial  $T$ . Now, the control polynomials  $S$

and  $R$  can be determined. After determining  $T$ , all the control polynomials will be known.

## 2 Model – Following

The Model – following condition will be used to determine control polynomial  $T$ . To be able to do so, the response from the command signal  $u_c$  to the output has to be described by formula

$$A_m y_m(t) = B_m u_c(t) \quad \text{--(11)}$$

Comparing formula , the condition written in formula must hold.

$$\frac{BT}{AR + BS} = \frac{BT}{A_c} = \frac{B_m}{A_m} \quad \text{--(12)}$$

Using some causality conditions, a minimum-degree solution of the Diophantine equation can be obtained. This means that there is always a solution for  $S$ . In the situation of model-following without zero cancelling, the control parameters become

Now the control polynomials  $R$ ,  $S$  and  $T$  can be determined. The solutions of these polynomials are the output of the part 'Control design' and input for part 'Controller' of fig 2 . With the earlier discussed estimator, all parts of the STR are explained

### 5.2.3 Algorithm Using RIs And Md Pole Placement

Off-line Parameters: Given polynomials  $B_m(q)$ ,  $A_m(q)$ ,  $A_o(q)$

Step 1: Estimate the coefficients of  $A(q)$  and  $B(q)$ , i.e. , using the Recursive Least Squares algorithm.

Step 2: Apply the Minimum Degree Pole Placement algorithm to compute  $R(q)$ ,  $T(q)$ ,  $S(q)$  with  $A(q)$ ,  $B(q)$  taken from the previous Step.

Step 3: Compute the control variable by  $R(q)u(t) = T(q)u_c(t) - S(q)y(t)$

Repeat Steps 1, 2, 3

## VI. SIMULATION RESULTS:

Simulations are carried out using MATLAB/SIMULINK

The open loop response obtained in simulation are shown in fig.4

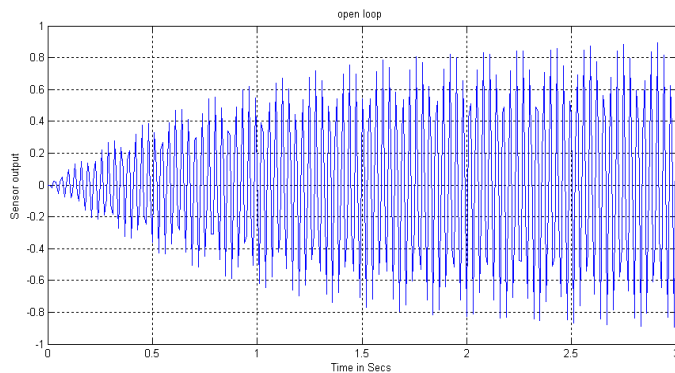


Fig 4 Open Loop Response

The PID controller suppresses the vibration about 80% in its amplitude are shown in fig. 5

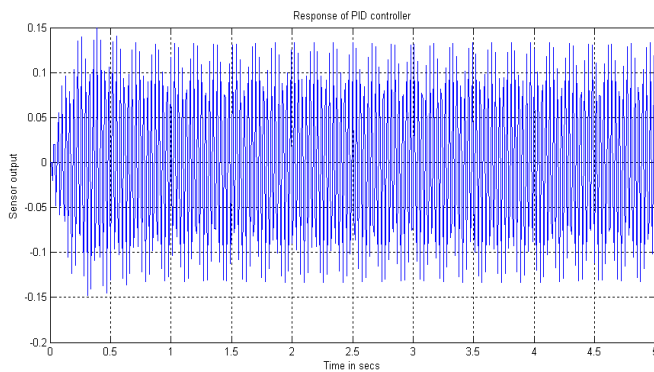


Fig 5 PID Controller Response

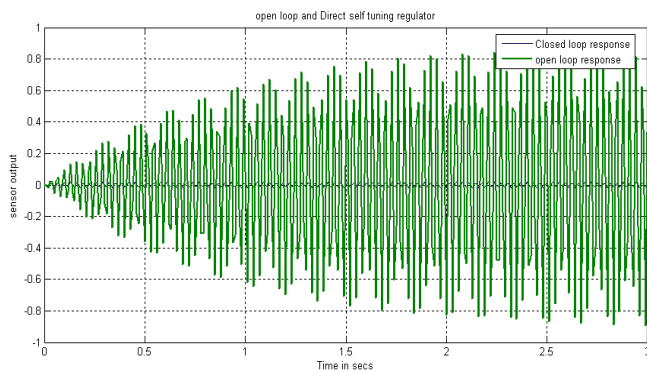


Fig 6 Response of Direct Self tuning controller

Adaptive controller suppresses the vibration about 98% in its amplitude using direct self tuning regulator shown in fig 6. and fig.7 shows the control input of the adaptive controller. The response of Indirect Self Tuning Regulator is shown in fig 8.

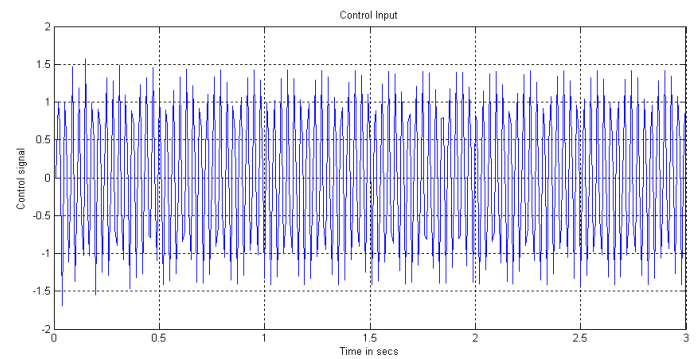


Fig 7 Control Input

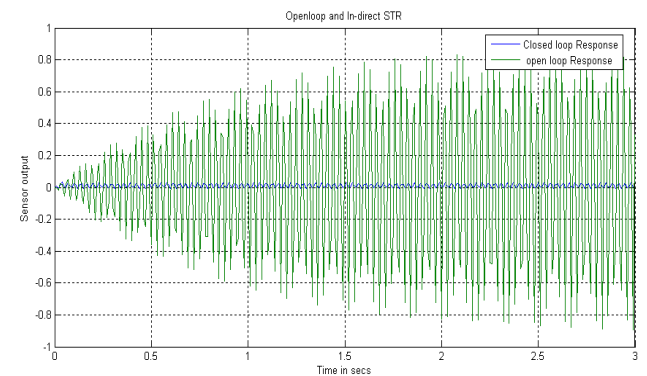


Fig 8 Response of In-Direct Self tuning controller

It is observed from these figures that the adaptive control system gives good vibration suppression results no matter whether the disturbances are applied transiently and continuously. It is observed that adaptive Self tuning control has satisfactory results no matter whether excitations are applied transiently and continuously. The Results are compared as follows in Table 2.

Table 2 The Results are compared in the table

Sl. No	Controller	Peak To Peak Amplitude	vibration suppressed in percentage
1	Open loop	1.8	-
2	PID	0.36	80
3	Adaptive – Direct STR	0.032	98.2
	Adaptive – In-Direct STR	0.034	98.1

## VII. CONCLUSION

Adaptive controller has adjusting mechanisms which adapts non-linearities present in the system. Compare to conventional PID Controller vibration suppression is effective with adaptive controller. Simulation results show that both the adaptive direct and In-direct self tuning control are effective vibration control schemes. Research is continuing to extend these results to real-time experiments and more complicated systems.

## REFERENCES

- [1] R. Maheswari, M. Umapathy, L. R. Karalmarx, D. Ezhilarasi, "Design and Implementation of Output Feedback Control for Piezo Actuated Structure Using Embedded System" *Sensors & Transducers Journal*, Vol. 93, Issue 6, June 2008, pp. 29-36
- [2] J. Fei, Y. Fang "Modeling and Adaptive Output Feedback Control for a Flexible Structure" *Proceedings of the 38th Southeastern Symposium on System Theory Tennessee Technological University Cookeville, TN, USA, March 5-7, 2006*
- [3] Zhang Guoqi, Liu jie, Wu Hongxin "Adaptive Control of Large Flexible Structures Using the Characteristic Modeling Technique" *IMACS Multiconference on "Computational Engineering in Systems Applications"(CESA)*, October 4-6, 2006, Beijing, China.
- [4] J. Fei "Adaptive Sliding Mode Vibration Control Schemes for Flexible Structure System" *Proceedings of the 46th IEEE Conference on Decision and Control New Orleans, LA, USA, Dec. 12-14, 2007*
- [5] Juntao Fei "Adaptive Vibration Control Schemes for Flexible Structure" *Proceedings of the 2007 IEEE International Conference on Mechatronics and Automation August 5 - 8, 2007, Harbin, China.*
- [6] Tamara Nestorovic' Trajkov \*, Heinz Ko"ppe, Ulrich Gabbert "Direct model reference adaptive control (MRAC) design and simulation for the vibration suppression of piezoelectric smart structures ", *Communications in Nonlinear Science and Numerical Simulation* 13 (2008) 1896–1909.
- [7] Karl Johan Astrom, Bjorn Wittenmark "Adaptive Control " (2nd Edition) Addison Wesley publishing company ,1995.