Denoising Various Imaging Modalities using Wavelets, Ridgelets and Curvelets: A Comparative Analysis

Mary Sugantharathnam D[1], Dr.Manimegalai.D[2]

[1]Dept. of ECE, Government College of Engineering, Tirunelveli, India
[2]Dept. of IT, National Engineering College, Kovilpatti, India
Email: [1]maryvictorin@yahoo.co.in

Abstract –

Image De-noising is a problem of prime importance in the field of Image Processing ranging from Medical Imaging to Satellite imaging. The main purpose of an Image de-noising algorithm is to reduce the noise level to improve both the interpretability and visual aspect of the images. This paper propose to indicate the suitability of different Multi-resolution transforms, viz Wavelets, Ridgelets and Curvelets in de-noising various imaging modalities corrupted by Random noise, Gaussian noise, Speckle Noise, Salt and Pepper Noise and Poisson noise. Though the comparative study is based on various imaging modalities, due relevance is given to medical images like Computed Tomography (CT), Magnetic resonance Imaging (MRI) and X-ray images. Experiments are conducted on various image data sets namely Natural, Satellite and Medical Images with the Multi-resolution transforms using two existing thresholding strategies, namely Soft and Hard Thresholding. A comprehensive evaluation of three different types of Multiresolution transforms corrupted with different types of noise is provided and the quality of de-noising is measured in terms of Peak Signal to Noise ratio (PSNR). Experimental results indicate that the Curvelets reveal superior performance over wavelets and Ridgelets in terms of PSNR value and perceptible quality.

Keywords **:**Curvelet, De-noising, Ridgelet, Threshold, Wavelet

I. INTRODUCTION

Efficient representation of images is critical for image processing in computer vision, pattern recognition and image compression. Often an image is corrupted by noise during acquisition as well as transmission, for instance in astronomical image, medical images and remote sensing image etc. Dealing with noisy data turns out to be one of the toughest challenges in image processing. Noise naturally arises in difficult conditions such as poorly illuminated environments, short exposure times and low efficiency photon detectors. In this context de-noising can help recover the underlying signal. In bio-medical applications, quality requirements are high and optimal de-noising increases both the readability and interpretability of the images (1). A typical example occurs in radiation oncology, where Positron Emission Tomography (PET) is used to diagnose tumors. Noise and resolution limit the accuracy of tumor delineation, which can reduce the treatment benefit. A common pre-processing step is de-noising, which is usually done via Gaussian smoothing. Smoothing suppresses noise, but it also changes the intensity variations of the underlying image. This suppresses or even removes the detailed features of the original image. The image corrupted by noise is not easily eliminated in image processing. According to actual image characteristics, noise statistical property and frequency

Spectrum distribution, noise elimination methods are divided into space and transformation fields. The space field is a data operation carried on the original image and processes the image grey value like Wiener filter etc. In the transformation field, the co-efficient after transformation are processed.

In the last decade, image denoising techniques have been increased especially for medical applications wherein the qualified radiologists navigate, view, analyze and interpret medical images. The analysis and visualization of the image stack received from the acquisition devices are difficult to evaluate due to the quantity of clinical data and the amount of noise existing in the medical images due to the scanners itself.Computed Tomography is one of the most important modalities in medical imaging and the radiation exposure associated with it is its drawback (2),(3). With respect to patients care, the least possible radiation dose is demanded. The ratio between relevant tissue contrasts and amplitude of noise must be sufficiently large for a reliable diagnosis. Single Photon Emission Computed tomography (SPECT) imaging is considered to be highly useful in oncology, but low Signal to Noise ratio (SNR)caused by photon noise introduces considerable compromise in image quality and reduction of diagnostic accuracy (4).Noise in medical X-ray images is primarily categorized into quantum mottle, which is related to the number of incident X-rays, and due to the artificial noise which is introduced due to the grid. The effect of quantum mottle is manifested as an increase in the graininess of an X-ray image as the dose is reduced. Therefore noise reduction is of great significance in medical X-ray images (5).

Hence sparse representation of image data, where most of the information is packed into small number of data, is very important in many image processing applications. In Image processing Fourier transform is usually used. However Fourier transform can only provide an efficient representation for smooth images but not for images that contain edges. Edges or boundaries of objects cause discontinuities or singularities in image intensity.

Wavelets are suitable for dealing with objects with point singularities. By decomposing the image into a series of high pass, low pass filter bands, the wavelet transform extracts directional details that capture horizontal, vertical and diagonal activity. Wavelets provide a very sparse and efficient representation for smooth signal, but it cannot efficiently represent discontinuities along edges or curves in images or objects (6).

Ridgelet improves de-noising; however they capture structural information of an image based on multiple radial directions in the frequency domain. Line singularities in Ridgelet transform provides better edge detection than wavelet. Ridgelet transform provides information about orientation of linear edges in images since it is based on Radon transform which is capable of extracting lines of arbitrary orientation. Ridgelet is most effective in detecting linear radial structures, hence not suitable for de-noising medical images.

Hence Donoho and others proposed the first generation Curvelet transform based on multiscale Ridgelet transform (7),(8). Later they proposed the second generation Curvelet transform. Two digital implementations of the Curvelet transform i.e. the Unequally Spaced Fast Fourier Transform (USFFT)[9] and the Wrapping Algorithm(10) are used to de-noise images degraded by different types of noise. Curvelet is proven to be particularly effective along curves which are the most comprising objects of medical images.

Although exhaustive comparative study on the performance of three different Multi -resolution transforms in de-noising various imaging modalities is projected in this paper much significance is given to Bio-medical images like CT, MRI and X-ray images since de-noising aids the physicians in a more accurate diagnosis of disease. A very important requirement for any noise reduction in medical images is that all clinically relevant images content must be preserved. Especially edges and small structures should not be affected. The goal of all these methods is to lower the noise power without smoothing curve edges.

Threshold shrinking algorithms are widely used in image de-noising. Structures are represented in a small number of dominant co-efficient while white noise is spread across a range of small co-efficient. This observation dates back to the work of Donoho and Johnston (11). The larger co-efficient are viewed as actual image signal while the smaller co-efficient are viewed as noise signal. So a proper threshold is set and if the co-efficient are smaller than the threshold it is set to zero, otherwise it will be kept unchanged(hard threshold) or shrunk in the absolute value by an amount of threshold(soft threshold). The thresholds are determined to a large extent by noise standard variance.

This paper is focused on robust implementation of Multi resolution analysis (MRA) techniques such as Wavelets, Ridgelet and Curvelets for denoising natural images, satellite images and medical images such as CT, MRI and X-ray images corrupted with different types of noise (5).

The rest of the paper is organized as follows. In Section 2, the analysis of Multi-resolution transforms are illustrated. The Image denoising algorithm is discussed in Section 3 and the Evaluation criteria is described in

Section 4. The experimental results and the Observations are discussed in Section 5. Finally the conclusions are drawn in Section 6. The future work in removing Poisson noise in medical images is highlighted in section 7.

II. METHODOLOGY-MULTIRESOLUTION ANALYSIS

Image De-noising using MRA such as wavelets has been widely used in recent years and provides better accuracy in de-noising different types of images. Many recent developments in MRA suggest Ridgelet and Curvelets since wavelets are suitable for dealing with objects with point singularities. Wavelets can only capture limited directional information due to its poor orientation selectivity. However the horizontal, vertical and diagonal activity extracted by the wavelet transform might not capture enough directional information in noisy images such as medical CT scans.

Hence Jean Luc Starck et al have proposed the radon, Ridgelet and curvelet transforms for image de-noising (12),(13). Ridgelets capture structural information of an image based on multiple radial directions in the frequency domain. Donoho and others proposed Curvelet transform and their anisotropic character which is particularly effective at detecting image activity along curves instead of radial directions which are the most comprising objects of medical images.

In this study comparative analysis of three different Multiresolution transforms (i.e.) Wavelets, Ridgelet and Curvelets using Soft and Hard thresholding have been proposed (14),(15). Second generation Curvelets (i.e.) the Fast Discrete Curvelet Transform based on wrapping algorithm is implemented as it is conceptually simpler, faster and less redundant.

I. *Wavelet Transform*

Wavelets are mathematical functions that analyze data according to scale or resolution. They aid in studying a signal in different windows or at different resolutions. For instance, if the signal is viewed in a large window, gross features can be noticed, but if viewed in a small window, only small features can be noticed. Wavelets do a good job than Fourier transform, in approximating signals with sharp spikes having discontinuities. The term “wavelets” is used to refer to a set of orthonormal basis functions generated by dilation and translation of scaling function φ and a mother wavelet ψ. The finite scale multi resolution representation of a discrete function can be called as a Discrete Wavelet Transform (DWT). DWT is a fast linear operation on a data vector, whose length is an integer power of two. This transform is invertible and orthogonal, where the inverse transform expressed as a matrix is the transpose of the transform matrix.

The orthonormal basis or wavelet function $\Psi_{(j,k)}(x)$is defined as

$$\Psi_{(j,k)}(x) = 2^{j/2}\Psi\left(2^j x - k\right)\Psi_{(j,k)}(x) = 2^{j/2}\Psi\left(2^j x - k\right) \qquad [1]$$

In equation [1]$2^{j/2}$ is the normalizing constant. The scaling function can be determined by equation [2] as follows

$$\phi_{j,k}(x) = 2^{j/2}\phi\left(2^j x - k\right) \qquad [2]$$

The factor ‘j’ in equations [1] and [2] is known as the scale index, which indicates the wavelet’s width. The location index *k* provides the position. The wavelet function is dilated by powers of two and is translated by the integer *k*. In terms of the wavelet coefficients, the wavelet equation is

$$\Psi(x) = \sum_{k}^{N-1} g_k\sqrt{2\phi(2x-k)} \qquad [3]$$

In equation [3], the term $\Psi(x)$ is the wavelet function and the coefficient g_k is the k^{th} high pass wavelet coefficient. Writing the scaling equation in terms of the scaling coefficients as given below, we get

$$\varphi(x) = \sum_{k}^{N-1} h_k\sqrt{2\phi(2x-k)} \qquad [4]$$

In equation [4], the term φ(x)is the scaling function and the coefficient $h_k$is the k^{th} low pass scaling coefficients. The wavelet and scaling coefficients are related by the quadrature mirror relationship, which is given by

$$g(n) = (-1)^{1-n}h(1-n) \qquad [5]$$

The term *N* is the number of vanishing moments. A graphical representation of DWT is shown in fig. 1. Note that, Y_0 is the initial signal.

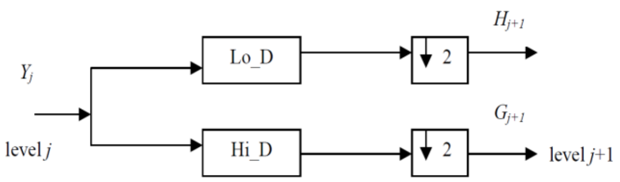

Fig.1. 1-Dimensional DWT - Decomposition step

The wavelet equation produces different wavelet families like Daubechies, Haar, Coiflets, etc. Wavelets are classified into a family by the number of vanishing moments *N*. Within each family of wavelets there are wavelet subclasses distinguished by the number of coefficients and by the level of iterations.

II. Ridgelet Transform

Ridgelet detect objects with line singularities. The Finite Ridgelet Transform can be determined by calculating the discrete Radon transform followed by the application of a Wavelet Transform. Computation of 2D Fast Fourier Transform for the image and application of 1D Inverse Fast Fourier Transform on each of the 32 radial directions gives the Finite Radon Transform (FRAT). The FRAT is used to map the image space to projection space since the set of projections of the image are taken at wide range of angles. A projection in discrete images means summation of all data points that lies within the specified unit-width strips.The FRAT of a real function on the finite grid Z_p^2 *is* defined as

$$r_k[l] = FRAT_f(k,l) = \frac{1}{\sqrt{p}} \sum_{(i,j)\in L(k,l)} f(i,j) \qquad [6]$$

Here, L (k, l) denotes the set of points that make up a line on the lattice Z_p^2 as in

$$L(k,l) = \{(i,j): j = k_i + l(\text{mod}p), i \in z_p\}, \quad 0 \le k < p \qquad [7]$$

$$L(p,l) = \{(i,j): j \in z_p\} \qquad [8]$$

Phistograms are to be used and all the pixels of the original image need to be passed once in order to compute the K[th] radon projection (16). Each output of the radon projection is simply passed through the wavelet transform before it reaches the output multiplier. As shown in fig. 2, Ridgelet use FRAT as a basic building block, where FRAT maps a line singularity into point singularity, and the wavelet transform has been used to effectively detect and segment the point singularity in radon domain.

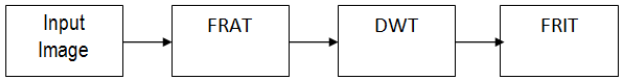

Fig. 2. FRIT block diagram

III. Curvelet Transform

Digital Curvelet Transform can be implemented using Fast discrete Curvelet Transform (FDCT) via USFFT and FDCT via Wrapping. These two algorithms differ by spatial grid used to translate Curvelets at each scale and angle. Curvelet Transform has a tight frame and combines multi scale analysis and geometrical ideas to achieve optimal rate of convergence by simple thresholding. It has strong directional character in which elements are anisotropic at fine scales. The support of these elements is according to the parabolic scaling principle length$^2\simeq$width.

<u>Fast Discrete Curvelet Transform via Wrapping</u>

In FDCT based on Wrapping of Fourier samples, an image of size M X N is taken as an input, where M and N are the dimensions and the outputs will be a collection of Curvelet co-efficient $C^D(j,l,k_1,k_2)$ indexed by a scale j, an orientation l and spatial location parameters k_1 and k_2. 'D' stands for Digital.

$$C^D(j,l,k_1,k_2) = \sum_{\substack{0\le m<M \\ 0\le n\le N}} \left[f(m,n)\varphi^D j,l,k_1,k_2[m,n]\right] \qquad (9)$$

In equation (9) the term f(m,n) is a Cartesian array, where $0 \leq \mathrm{m} < M, 0 \leq n \leq N$ where M and N are dimensions of the array. Wrapping based Curvelet Transform is a multiscale pyramid which consists of several subbands at different scales consisting of different orientations and positions in the frequency domain. Curvelets are very fine at high frequency level and at low frequencies they are non-directional coarse elements.

In general curvelet transform can be accomplished in the frequency domain, to attain high efficiency. Hence two dimensional Fast Fourier Transform (2D FFT) is applied to the image to convert it from spatial domain to frequency domain prior to curvelet transform. For each scale and orientation, a product of U_{jl}"wedge" is obtained; the result is then wrapped around the origin. Finally a two dimensional Inverse Fast Fourier Transform (2D IFFT) is then applied which yields in discrete curvelet coefficients. The discrete curvelet transform can be determined as

Curvelet transform = IFFT [FFT (Curvelet) × FFT (Image)] [10]

The difficulty behind this is that trapezoidal wedge does not fit in a rectangle of size $2^j \times 2^{j/2}$ aligned with the axes in the frequency plane in which the 2D IFFT could be applied to collect curvelet coefficients. Wedge wrapping procedure uses a parallelogram with sides 2^j and $2^{j/2}$ to support the wedge data. The wrapping is achieved by periodic tiling of the spectrum inside the wedge and then collecting the rectangular coefficient area in the center. The center rectangle of size $2^j \times 2^{j/2}$ successfully collects all the information in that parallelogram.

The steps of applying wrapping based FDCT algorithm are as follows.

Step 1: The 2D FFT is applied to an image to obtain Fourier samples $\hat{f}[m,n], -\frac{n}{2} \leq m,n < \frac{n}{2}$

Step 2: For each scale j and angle l, the product $\widetilde{\mathrm{U}}_{\mathrm{j,l}}[\mathrm{m,n}]\hat{\mathrm{f}}[\mathrm{m,n}]$ is formed.

Step 3: Wrap this product around the origin to obtain $\tilde{\mathrm{f}}_{\mathrm{j,l}}[\mathrm{m,n}] = \mathrm{W}(\widetilde{\mathrm{U}}_{\mathrm{j,l}}\hat{\mathrm{f}})[\mathrm{m,n}]$ where the range for m, n, and θ is now $0 \leq m < 2j$, $0 \leq n < 2^{j/2}$, and $-\pi/4 \leq \theta < \pi/4$.

Step 4: Apply IFFT to each $\tilde{\mathrm{f}}_{\mathrm{j,l}}$ and hence collect the discrete coefficients $\mathrm{C^D}(\mathrm{j,l,k_1,k_2})$.

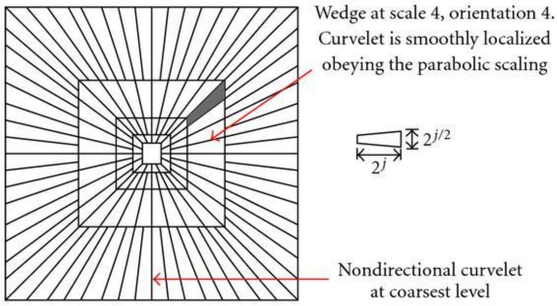

Fig 3. Curvelet Tiling of an image in the spectral domain (5levels)

Fig.3 shows the combined frequency response of Curvelets at different scales and orientations in the form of rectangular tiling of an image. As the resolution level is increased, the Curvelet is more sensitive to curved edges and hence curves in an image are represented well.

IV. IMAGE DENOISING

Image De-noising is used to produce good estimates of the original image from noisy observations. The restored image should contain less noise than the observations while still keep sharp transitions (i.e. edges).Suppose an image f(m,n) is corrupted by the additive noise

$$g(m,n) = f(m,n) + \eta(m,n) \qquad [11]$$

where η(m,n) are independent identically distributed Gaussian random variable with zero mean and variance σ^2.Image de-noising algorithms vary from simple thresholding to complicate model based methods. However simple thresholding methods can remove most of the noise. Various types of noise used in are Salt and Pepper Noise, Gaussian noise, Random noise, Speckle noise and Poisson noise. The Gaussian noise is independent at each pixel and independent of signal intensity .Random noise is characterized by intensity and color fluctuations above and below the actual image intensity. The pattern of random noise changes even if the exposure settings are identical. Random noise revolves around an increase in the intensity of the picture and as such it can be said that random noise are hundred folds more than other types of noise.

A. Algorithm

The following steps is applicable to Wavelet Transform, Ridgelet Transform and Curvelet Transform

1. Apply the Forward transform to the noisy image.
2. Threshold the co-efficient to remove some insignificant co-efficient by using a thresholding function in the corresponding transform domain.
3. Take Inverse transform of the thresholded co-efficient to reconstruct a function.

B. Thresholding Function

1. Soft Thresholding is defined by a fixed threshold$\sigma > 0$

$$S_\sigma(x) = \begin{cases} x - \sigma & x \geq \sigma \\ 0 & |x| < \sigma \\ x + \sigma & x \leq \sigma \end{cases} \quad [12]$$

2. Hard Thresholding

$$S_\sigma(x) = \begin{cases} x & |x| \geq \sigma \\ 0 & |x| < \sigma \end{cases} \quad [13]$$

These thresholding functions might be a good choice because large co-efficient remain nearly unaltered.

V. EVALUATION CRITERIA

To compare the results of different curvelet thresholding techniques, a image similarity measure is used.

$$\mathrm{PSNR} = 10 \log_{10}\left(\frac{f_{max}^2}{\mathrm{MSE}}\right) \quad [14]$$

In equation (14) the term f_{max} is the maximum value of the image intensities and MSE is the mean square error between the reconstructed image and the original image.

$$\mathrm{MSE} = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|f(m,n) - \tilde{f}(m,n)\right|^2 \quad [15]$$

In equation [15] the term f(m,n) is the original image and the term $\tilde{f}(m,n)$ f˜(m, n is the de-noised image. M x N is the number of pixels. The de-noised image is closer to the original one when PSNR is higher.

VI. RESULTS AND DISCUSSIONS

In this section we present experimental results that demonstrate properties of Wavelets, Ridgelet and Curvelets and its potential applications. The experiments are conducted on several types of gray scale images of size 256 x 256 using MATLAB.

CT scan images and MRI scan images of brain slice, X-ray images, Natural images and Satellite images are de-noised using Wavelets, Ridgelets and Curvelet transform using Wrapping Technique. Various types of noise namely Random noise, Additive white Gaussian noise, Speckle noise, Salt and Pepper and Poisson noise are added. Simulations were carried out for each of these transforms for different noise variance levels.

Inspired by sparse representation of Multiresolution transforms (i.e.) Wavelets, Ridgelet and Curvelets are applied to image de-noising. Haar wavelet with single level of decomposition is used. It is assumed that the noise variances were known in order to focus on the denoising techniques themselves. Thresholding function for the various transform domains is used.

The Curvelet transform can achieve higher PSNR than Ridgelets and Wavelets and the Curvelet transform can restore edges better than Ridgelets or Wavelets. Edges are blurred due to thresholding of co-efficient in the wavelet domain. Ridgelet transform can be used in applications where images contain edges and straight lines. Ridgelet is effective in detecting linear radial structures which are not dominant in medical images.

The average PSNR gain of various gray scale images degraded by Random noise, Salt & Pepper noise, Speckle noise, Gaussian noise and Poisson noise with Wavelets, Ridgelets and Curvelets are listed in Tables 1 to 5(17).

The de-noising results for various gray scale images corrupted with Random noise, Salt and Pepper noise, Gaussian noise, Speckle and Poisson noise are shown in figures 4 to 9. In this experiment three types of Multiresolution transforms namely Wavelets, Ridgelet and

Curvelets based on soft and hard thresholding are applied to several gray scale imaging modalities and the efficiency of a particular Multiresolution transform in denoising a specific imaging modality is justified in terms of PSNR.

A. Comparison on Natural Images

Experiments were conducted on 3 standard test images 'Lena', 'Barbara' and 'pout' of size 256x256 pixels. The result shows that the average PSNR gain achieved with Curvelets is the highest when compared to Ridgelets and Wavelets for images contaminated with Salt and Pepper Noise, Gaussian Noise, Speckle Noise and random noise and Poisson noise implemented with soft and hard thresholding. The results are listed in table 1. However it is observed that the PSNR gain obtained with wavelets is higher with soft thresholding in the removal of Poisson noise.

B. Comparison on Bio-Medical images

The experiment was simulated on 3 test images each, CT, MRI and X-ray of size 256x256 pixels. Curvelets outperform Wavelets and Ridgelets in the removal of Salt and Pepper noise, Gaussian Noise, Speckle noise and Random noise with the exception that the Ridgelet transform with hard thresholding is efficient in the removal of Salt and Pepper noise. It is also observed that Wavelets offers better results in the removal of speckle noise in MRI images.

A negative PSNR gain is obtained with the three Multiresolution transforms for images corrupted with Poisson noise which justifies that the existing nonlinear thresholding strategy offers poor results in the removal of Poisson noise in Bio- medical images.

The improvement in the removal of Poisson noise in X-ray images is very small of the order of about 0.5 dB with wavelets implemented with soft thresholding where Curvelets offer a gain of about 2.11db. The results are tabulated in tables 2, 3 and 5.

C. Comparison on Satellite Images

Experiments were conducted on three satellite images of size 256x256 pixels. Observations indicate that Curvelets implemented with soft and hard thresholding proves to be efficient in the removal of Salt and Pepper noise, Gaussian noise, speckle noise and random noise with the exception that the Ridgelet Transform with hard thresholding offers superior results in the removal of Salt and Pepper noise in Satellite images. Table 4 illustrates the performance of various de-noising techniques on satellite images.

D. Observations

Three different multiscale transforms viz Wavelets, Ridgelets and Curvelets were implemented to denoise various imaging modalities degraded by different types of noises using soft and hard thresholding. Experiments conducted on different data sets reveal the following facts.

1. The average PSNR gain increases for almost all types of imaging modalities as the noise variance increases.
2. As the number of decomposition levels increases in a wavelet, the average PSNR gain also increases.
3. Curvelets outperforms wavelets and Ridgelets in the removal of Salt and Pepper Noise and random noise with soft and hard thresholding with the exception that Ridgelets offer better results with hard thresholding in the removal of salt and pepper noise in various imaging modalities.
4. Speckle being an multiplicative noise, a negative gain is obtained with Ridgelets and Curvelets when denoising MRI images, whereas wavelets offers very poor gain ranging from .01 dB to 0.4 dB which is not much of an improvement to be mentioned.
5. Wavelets, Ridgelets and Curvelets offers inferior results in the removal of poisson noise in Bio-medical images like CT, MRI and X-ray using the non-linear thresholding strategy.

VI. CONCLUSION

In this paper a strategy for digitally implementing the Wavelets, Ridgelets and Curvelets to de-noise gray scale images degraded by Salt and Pepper Noise, Gaussian Noise, Speckle Noise, Random noise and Poisson noise is presented. The results of de-noising are measured in terms of PSNR and the comparison is based on the average PSNR gain as shown in tables 1 to 5. The graphs illustrate the results of the various Multiscale resolution transforms in terms of average PSNR gain. The experimental study has been implemented using the existing thresholding strategy (i.e.) soft and hard thresholding for noise removal.

Based on the results obtained it is concluded that Curvelet outperforms Wavelets and Ridgelets in de-noising different imaging modalities corrupted with Salt and Pepper Noise, Gaussian noise, Speckle noise, Random noise and Poisson noise and yields improved performance in terms of PSNR.

Experimental results indicate that the Curvelet Transform can provide a more efficient representation for images with singularities along smooth curves and is suitable for representing edges in images. Curvelet transform exhibits good reconstruction of the edge data by incorporating a directional component to the traditional wavelet transform.

The evaluation of the results supports the conclusion that curvelets has significantly better results than others. From the restored images it can be visually depicted that edges are well preserved taking the advantage of the fact that Curvelets being geometrical, very well align themselves to the contours of the edges. The reconstruction via curvelet transform does not contain the quality of disturbing artifacts along edges that is observed in wavelet reconstruction.

Thus Curvelet transforms enjoys superior performance over wavelets and ridgelets in the context of denoising.It is also observed that Wavelets, Ridgelets and Curvelets implemented with the existing thresholding strategy fails to remove Poisson noise in Biomedical images, though considerable improvement is obtained with Natural images and Satellite images. It is henceforth concluded that by using the existing thresholding strategy, all three multiresolution transforms fail to remove Poisson noise in Bio-medical images.

VII. FUTURE WORK

Future research will focus on finding a novel method for removing Poisson noise in medical images, especially in PET (Positron Emission Tomography) and SPECT data which uses Multi Scale Variance Stabilizing Transforms (MS-VST) (2) which combines the VST with the low pass filter involved in various multi scale multi direction transforms (MS-MD).

Table 1.AVERAGE PSNR GAIN (dB) OF WAVELETS, RIDGELET AND CURVELETS FOR NATURAL IMAGES

Noise	Noise Variance	Soft Thresholding			Hard Thresholding		
		Wavelet	Ridgelet	Curvelet	Wavelet	Ridgelet	Curvelet
Salt & Pepper	0.05	0.9253	1.4340	6.3000	0.0094	1.6800	2.4000
	0.08	1.1307	2.2990	7.1700	-0.0432	2.5420	2.0900
	0.2	2.8117	3.7370	7.3000	2.5813	3.8830	1.3600
Gaussian	0.01	2.6311	0.8930	7.0200	2.6391	1.1250	8.7000
	0.02	2.8258	1.0340	6.5000	2.8575	1.1970	3.9000
	0.2	2.8312	0.8810	0.6900	2.8461	0.6810	-2.7900
Speckle	0.02	2.4074	-0.7610	4.0700	2.4109	-0.4950	6.8700
	0.04	2.6694	0.6180	6.9800	2.6741	0.8850	8.5600
	0.08	2.8161	1.9030	9.5200	2.8012	2.1730	6.9100
Random	20	1.0449	-0.1051	5.9500	1.0381	0.1116	7.9000
	25	1.1061	1.2445	6.4037	1.1050	1.4536	9.0002
	30	1.1649	1.9204	7.4085	1.1440	2.1339	9.7406
Poisson	-	1.6839	-2.8073	0.6900	1.7420	-2.5317	3.3800

Table 2.AVERAGE PSNR GAIN (dB) OF WAVELETS, RIDGELET AND CURVELETS FOR CT IMAGES

Noise	Noise Variance	Soft Thresholding			Hard Thresholding		
		Wavelet	Ridgelet	Curvelet	Wavelet	Ridgelet	Curvelet
Salt & Pepper	0.05	0.4283	2.7960	4.9000	-0.0627	2.8030	1.9800
	0.08	0.5593	3.4500	5.3900	0.0156	3.4040	1.6500
	0.2	1.7997	4.1070	5.2900	0.1645	4.0020	1.2100
Gaussian	0.01	1.3526	1.1100	3.3000	1.4068	1.0730	6.2000
	0.02	1.7930	1.2130	3.2200	1.7947	1.1250	2.9900
	0.2	2.0781	0.8400	0.5500	2.1114	0.6570	-2.5600
Speckle	0.02	0.8966	-1.2180	-0.2600	0.4616	-1.0170	3.6800
	0.04	1.2733	0.0640	2.3000	0.6086	0.2370	5.2300
	0.08	1.5366	1.1940	4.3600	0.7104	1.3920	4.8700
Random	20	1.0196	0.0522	4.5100	1.1118	0.2602	7.5000
	25	1.6095	1.0374	4.7105	1.6475	1.2164	8.2324
	30	1.7990	1.7243	5.5380	1.8093	1.8854	8.9845
Poisson	-	-0.0555	-3.2722	-4.2000	0.2298	-3.0526	0.2500

Table 3.AVERAGE PSNR GAIN (dB) OF WAVELETS, RIDGELET AND CURVELETS FOR MRI IMAGES

Noise	Noise Variance	Soft Thresholding			Hard Thresholding		
		Wavelet	Ridgelet	Curvelet	Wavelet	Ridgelet	Curvelet
Salt & Pepper	0.05	0.0586	3.7230	5.1300	0.0312	3.6680	1.9800
	0.08	0.0850	4.1760	5.3700	-0.0356	4.1010	1.6700
	0.2	0.5337	4.1900	5.0000	-0.0323	4.0560	1.0500
Gaussian	0.01	1.1518	1.3140	2.7000	1.2319	1.2380	4.6000
	0.02	1.4960	1.3510	2.6700	1.4981	1.2270	2.6100
	0.2	1.7628	0.0810	0.6700	1.7637	0.5360	-1.7700
Speckle	0.02	0.3034	-2.9490	-5.3100	0.0125	-2.8990	-2.7100
	0.04	0.3409	-1.5350	-2.4000	0.0432	-1.4780	-0.0200
	0.08	0.4027	-0.1900	0.3100	0.0456	-0.1210	1.8600
Random	20	0.8851	0.7079	4.2000	0.9995	0.7072	6.4400
	25	1.3649	1.5346	5.4612	1.4019	1.5343	7.6282
	30	1.6852	2.2139	6.4439	1.7127	2.1995	8.6245
Poisson	-	0.0800	-4.4287	-8.3000	0.1174	-4.3757	-5.7000

Table 4.AVERAGE PSNR GAIN (dB) OF WAVELETS, RIDGELET AND CURVELETS FOR SATELLITE IMAGES

Noise	Noise Variance	Soft Thresholding			Hard Thresholding		
		Wavelet	Ridgelet	Curvelet	Wavelet	Ridgelet	Curvelet
Salt & Pepper	0.05	0.0344	5.3850	6.3000	0.0001	5.1020	2.4000
	0.08	0.0667	5.4710	6.3000	0.0675	5.2630	1.9000
	0.2	0.5262	4.9400	5.5000	-0.0343	4.6530	1.2000
Gaussian	0.01	1.9005	3.3570	5.4000	1.8747	3.1090	6.9000
	0.02	2.0472	3.2210	5.0000	2.0077	2.9520	3.5000
	0.2	1.9799	0.9200	0.8000	1.9783	0.7540	-2.0000
Speckle	0.02	0.6044	1.3540	2.1000	0.1060	1.5830	5.1000
	0.04	0.6674	2.6210	5.1200	0.1391	2.8150	7.7000
	0.08	0.6686	3.6940	7.9000	0.1168	3.8320	7.3000
Random	20	2.2492	3.5097	8.7000	2.2806	3.7687	11.2000
	25	2.4565	4.1449	9.8826	2.4566	4.3771	12.1821
	30	2.5897	4.6310	10.7962	2.5590	4.8537	13.0017
Poisson	-	0.6246	-0.2546	-1.4200	0.1923	-0.0565	1.7100

Table 5.AVERAGE PSNR GAIN (dB) OF WAVELETS, RIDGELET AND CURVELETS FOR X-RAY IMAGES

Noise	Noise Variance	Soft Thresholding			Hard Thresholding		
		Wavelet	Ridgelet	Curvelet	Wavelet	Ridgelet	Curvelet
Salt & Pepper	0.05	0.2510	3.1060	5.7790	-0.0520	3.1720	2.2860
	0.08	0.3633	3.6990	5.9800	0.0103	3.6470	1.5700
	0.2	1.5170	4.1360	5.4410	0.0506	4.0020	1.0660
Gaussian	0.01	1.4300	1.3230	5.1800	1.9200	1.3110	6.8900
	0.02	1.3600	1.4030	4.8300	1.7700	1.3440	6.2200
	0.2	0.2600	0.8170	0.7000	0.2100	0.6300	0.7900
Speckle	0.02	0.8380	-0.8010	6.3980	1.4660	-0.6280	9.5764
	0.04	1.8200	0.4280	7.3400	1.8400	0.5710	10.5000
	0.08	2.1000	1.4620	8.0800	2.0400	1.6220	11.3000
Random	20	1.4000	0.4484	2.4300	0.2400	0.5854	5.9300
	25	0.3000	1.2793	4.7500	0.3000	1.4140	6.6300
	30	1.7500	1.8922	6.3200	0.3100	2.0713	4.9800
Poisson	-	0.5100	-2.9882	-1.7700	0.6100	-2.8202	2.1100

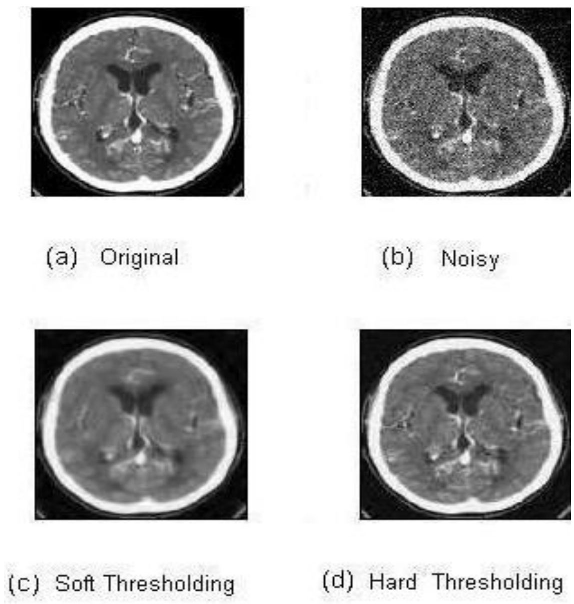

Fig. 4. Denoising results of curvelet transform with soft and hard thresholding (Gaussian Noise – variance = 0.01) (a) Original (b) Noisy (PSNR=20.7430dB) (c) Soft (PSNR=25.1597dB) (d) Hard (PSNR =27.8328dB)

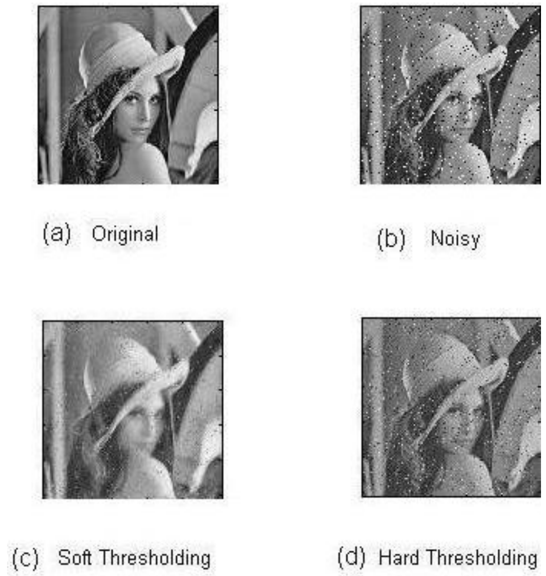

Fig. 5. Denoising results of curvelet transform with soft and hard thresholding(Salt & Pepper Noise-variance = 0.05) (a) Original (b) Noisy (PSNR=18.4051dB) (c) Soft (PSNR= 23.8509dB) (d)Hard (PSNR =20.6954dB)

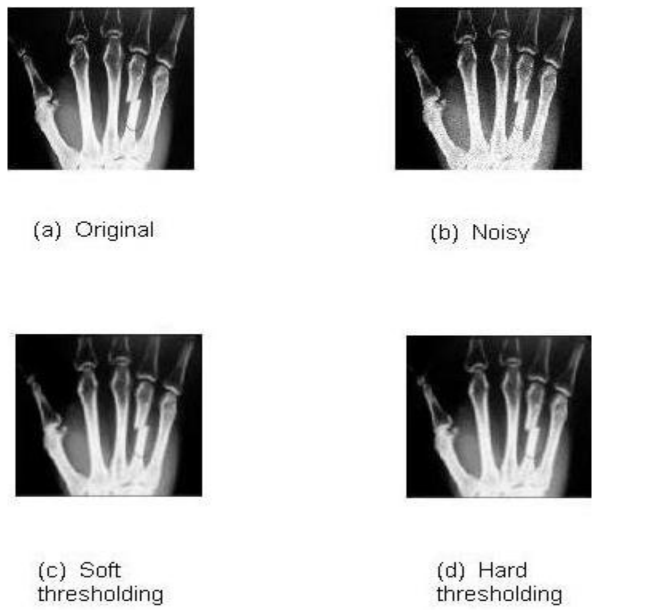

Fig. 6. Denoising results of Ridgelet transform with soft and hard thresholding (x ray -3 Poisson Noise) (a) Original (b) Noisy (PSNR=38.7764dB) (c) Soft (PSNR=35.7072dB) (d)Hard (PSNR =35.8809dB)

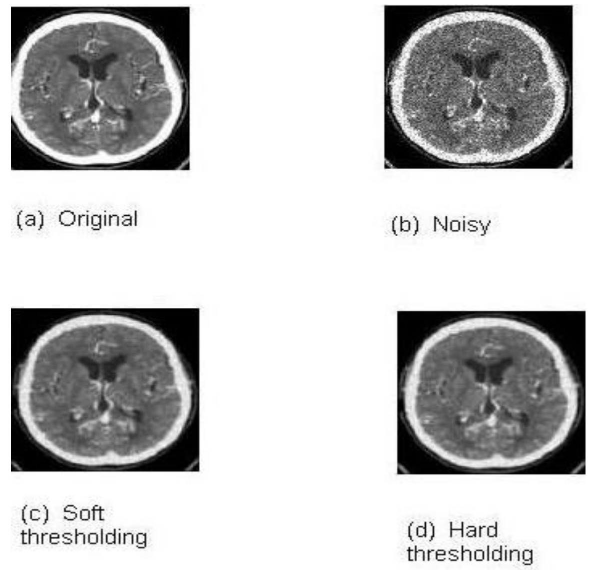

Fig. 7. Denoising results of curvelet transform with soft and hard thresholding (CT1 Speckle Noise: Variance = 0.04) (a) Original (b) Noisy (PSNR=34.7777dB) (c) Soft (PSNR= 35.0319dB) (d)Hard (PSNR =35.2520dB)

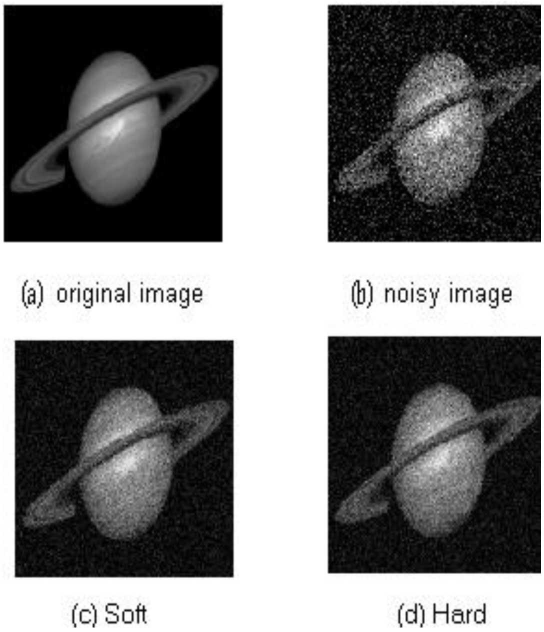

Fig. 8. Denoising results of Wavelet transform with soft and hard thresholding (Gaussian Noise: variance = 0.02) (a) Original (b) Noisy (PSNR= 33.5420dB) (c) Soft (PSNR=35.5685dB) (d)Hard (PSNR =35.5290dB)

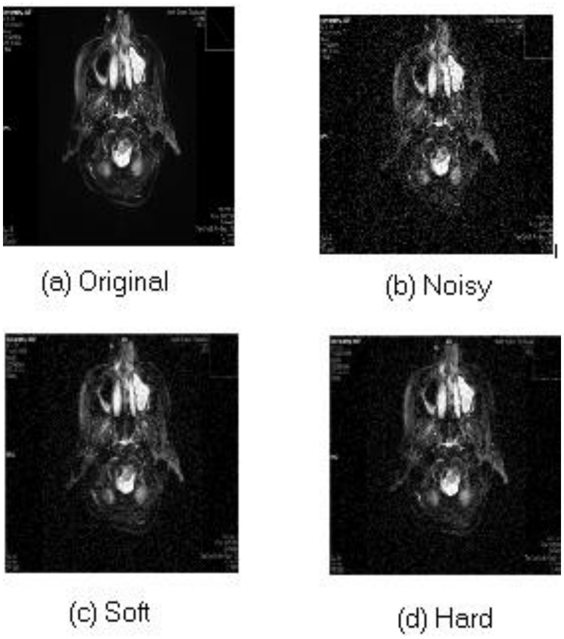

Fig. 9. Denoising results of Wavelet transform with soft and hard thresholding (Random σ=20) (a) Original (b) Noisy (PSNR=36.0586dB) (c) Soft (PSNR=36.8511dB) (d)Hard (PSNR =37.022dB)

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