International Journal on Information Sciences and Computing, Vol.2, No.1, July 2008

ALGEBRAIC CONSERVATIVE PETRI NETS BASED ON SYMMETRIC GROUPS

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Abstract

In this paper we define a new sub class of Petri nets called algebraic conservative Petri nets (ACPN) for a given symmetric group S_n. We prove that the resulting Petri net (ACPN) is a marked graph. In particular, we show that the algebraic conservative Petri nets associated with S3 and S5 has decompositions $\pi = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$ and $\pi' = \{\pi_1, \pi_2, \pi_3, \pi_4, \ldots, \pi_{84}\}$ respectively, for the sets of places such that each block π_i is both siphon and trap and hence the underlying directed graphs of these algebraic conservative Petri nets are Eulerian. Also we show that each of the ACPN associated with these groups has a subset of places which are both siphon and trap such that the input transitions equal the output transitions and both of them equal to the set of all transitions of these algebraic conservative Petri nets and hence that the underlying directed graphs of these algebraic conservative Petri nets associated with S_3 and S_5 are Hamiltonian.

Keywords: Algebraic Conservative Petri Nets, Siphons, Traps, Symmetric Groups , Directed Graphs

I. INTRODUCTION

Petri net is a mathematical modeling tool for concurrent systems and has been widely investigated by many researchers [1,2]. A Petri net consists of two kinds of nodes called places and transitions. Directed arcs are used to connect places to transitions and transitions to places. Small dots, called tokens in the places represent a marking of a Petri net. In a graphical representation of a Petri net the places are represented by circles and transitions are represented by bars or small boxes. Various areas of applications of Petri nets include modeling and analysis of the distributed systems, parallel processes, information systems, databases, communication protocols,

The study of structural properties and behavioral properties for marked graphs has been made utilizing siphons and traps [4,5,6]. A nonempty subset of places J is called a siphon if every transition having an output place in J has an input place in J. A nonempty subset of places Q is called a trap if every transition having an input place in Q has an output place in Q.

In this paper we define a new sub class of Petri nets called algebraic conservative Petri nets *(ACPN)* for a given symmetric group S_n. We prove that the resulting Petri net *(ACPN)* is a marked graph . In particular, for the groups S_3 and S_5 , we show that each of the *ACPN* associated with computer hardware architectures, manufacturing systems, formal languages and automata, learning theory and graph theory [5,7].

These groups has a subset of places which are both siphon and trap such that the input transitions equal the output transitions and both of them equal to the set of all transitions of these algebraic conservative Petri nets and hence that the underlying directed graphs of these algebraic conservative Petri nets are Hamiltonian. Also we show that the algebraic conservative Petri nets associated with S_3 and S_5 has decompositions $\pi = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$ and $\pi' = \{\pi_1, \pi_2, \pi_3, \pi_4, \ldots, \pi_{84}\}$ respectively, for the sets of places such that each block $_i$ is both siphon and trap and hence the underlying directed graphs of these algebraic conservative Petri nets associated with S_3 and S_5 are Eulerian.

II. PRELIMINARIES

In this section we present some basic definitions relevant to this paper.

Definition 2.1: A Petri net is triple $N = (P, T, F)$ where P is a finite set of places, T is finite set of transitions such that

(I) $P \cup T \neq \; ; P \cap T = \varphi$

(ii) $0 \; F \subseteq (P \times T) \cup (T \times P)$ is set of directed arcs.

For all $p \; P$, ${}^{\bullet}p = \{t \in T \mid (t, p) \in F\}$ and $p^{\bullet} = \{t \in T \mid (p, t) \in F\}$ be the input and output sets of p respectively. Similarly for all $t \in T$, ${}^{\bullet}t = \{p \in P \mid (p, t) \in F\}$ and $t^{\bullet} = \{p \in P \mid (t, p) \in F\}$ be the input and output sets of t respectively.

Definition 2.2: A Petri net is said to be a marked graph if $|{}^{\bullet}p| = |{}^{\bullet}p| = 1$ for all $p \in P$.

Definition 2.3: A Petri net is said to be conservative if $|{}^{\bullet}t| = |t^{\bullet}|$ for all $t \in T$.

Definition 2.4: A non-empty subset of places J in a Petri net is called a siphon if ${}^{\bullet}J \subseteq J^{\bullet}$. That is every transition having an output place in J has an input place in J.

Definition 2.5: A nonempty subset of places Q in a Petri net graph is called a trap if $Q^{\bullet} \subseteq Q^{\bullet}$. That is every transition having an input place in Q has an output place in Q.

Definition 2.6: A non empty subset Z of places in a Petri net graph is said to be both siphon and trap if $^{\bullet}Z = Z^{\bullet}$. That is, every transition having an input place in Z has an output place in Z and vice versa.

Example 2.7: Consider a Petri net shown in Fig. 1

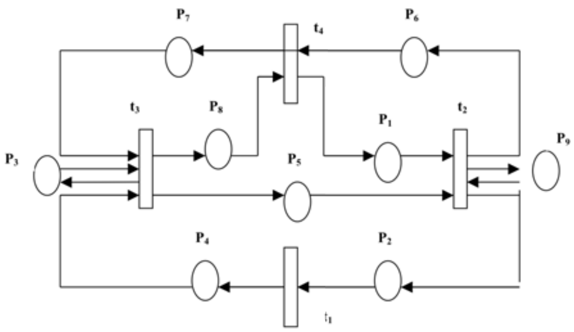

Fig .1 A Petri net

In this Petri net let $J = \{p_1, p_6, p_7\}$. Then $^{\bullet}J = \{t_2, t_4\}$, and $J^{\bullet} = \{t_2, t_3, t_4\}$. Here $^{\bullet}J \subseteq J^{\bullet}$. Therefore J is a siphon. Let $Q = \{p_6, p_7, p_8\}$. Then $^{\bullet}Q = \{t_2, t_3, t_4\}$, $Q^{\bullet} = \{t_3, t_4\}$. Here $Q^{\bullet} \subseteq {}^{\bullet}Q$. Therefore Q is a trap.

Let $Z = \{p_2, p_4, p_5\}$. Now, $^{\bullet}Z = \{t_1, t_2, t_3\}$, $Z^{\bullet} = \{t_1, t_2, t_3\}$. Here $Z^{\bullet} = {}^{\bullet}Z$. Therefore Z is both siphon and trap.

Definition 2.8: The symmetric group S_n is the group of all permutations on n symbols. This S_n is called a permutation group of order $n!$

Example 2.9: Consider the group $S_3 = \{\alpha, \beta, \gamma, \delta, \varphi, \Psi\}$ where $\alpha = (1)(2)(3)$, $\beta = (13)(2)$, $\gamma = (12)(3)$, $\delta = (132)$, $\varphi = (1)(23)$, $\Psi = (123)$ with generating set $S = \{\alpha, \Psi\}$

III. ALGEBRAIC CONSERVATIVE PETRI NETS

In this section we define the new sub class of Petri nets and prove the main results.

Theorem 3.1: There exist a algebraic conservative Petri net for every symmetric group with a generating set.

Proof: Let S_n be a symmetric group with generating set $S = \{(1, 2), (1, 2, 3,...,n)\}$. Let $\gamma = (1, 2)$ and $\delta = (1, 2, 3,...,n)$. Take the elements $t_1, t_2, t_3, t_4, \ldots, t_{n!}$ of the group S_n as transitions and set $T = \{t_1, t_2, t_3, t_4, \ldots, t_{n!}\}$. Now $S \subseteq T$. For every $t_i \in T$ and $t_k \in S$ such that $t_i t_k = t_j$ introduce a place p such that $^{\bullet}p = t_i$ and $p^{\bullet} = t_j$. Since the generating set S has 2 elements and the group has $n!$ elements, this process will yield a Petri net N with place set P and the transition set T such that $|T| = n!$ and $|P| = 2. n!$. Since the set S has exactly two elements, we have that each transition has exactly two input places and exactly two output places. Now keep tokens in a place p if p is the input of $t_i . t_j$ for every $t_i, t_j \in S$. Thus we have constructed a conservative Petri net with initial marking. The resulting Petri net on S_n with generating set as S is called algebraic conservative Petri net denoted as $ACPN(S:S_n)$.

Proposition 3.2: Every algebraic conservative Petri net ($ACPN$) for the symmetric group of order $n!$, with a generating set is a marked graph.

Proof: From the construction of algebraic conservative Petri net, (Theorem 3.1) the places are introduced in such way that each place has exactly one input transition and exactly one output transition. Hence by definition, it is a marked graph.

Lemma 3.3 The algebraic conservative Petri net on S_n is bounded but not 1-safe.

Proof: From the construction of algebraic conservative Petri net, a token is deposited in a place p if p is the input of $t_i . t_j$ for every $t_i, t_j \in S$. So, initially $n!$ places will receive tokens. Since the this Petri net is conservative, the tokens deposited in this net is neither created nor destroyed. Hence any place can have a maximum of $n!$ tokens and thus it is bounded. Again, there is a possibility of a place which can have $n!$ tokens in it, we conclude that it is not 1 – safe.

The construction of underlying directed graph Ns_n for the given marked graph N is given in [4,5].

Theorem 3.4: If the $ACPN(S:S_n)$ for a given symmetric group S_n has a subset of places Z such that $^{\bullet}Z = Z^{\bullet} = T$ where T is the set of all transitions of $ACPN(S:S_n)$, then the underlying directed graph Ns_n has a Hamiltonian circuit.

Proof: In [4] it is proved that if there exist a subset Z of places in a marked graph such that $^{\bullet}Z = Z^{\bullet} = T$, where T is the set of all transitions of the marked graph then the edges corresponding to the places in Z constitute a directed Hamiltonian circuit in the underlying directed graph. Hence by proposition 3.2, the theorem follows.

Theorem 3.5: If the place set P of $ACPN(S:S_n)$ for a given symmetric group S_n has a decomposition $\pi = \{\pi_1, \pi_2, \pi_3, \ldots, \pi_n\}$ in which each block $_i$ both siphon and trap then the underlying directed graph Ns_n of $ACPN(S:S_n)$ is Eulerian.

Proof: In [5] it is proved that if there exist a decomposition $\pi = \{\pi_1, \pi_2, \pi_3, \ldots, \pi_n\}$ for the set of places of a marked graph such that each block $_i$ in the decomposition $\pi = \{\pi_1, \pi_2, \pi_3, . . ., \pi_n\}$ is both siphon and trap then the underlying directed graph of that marked graph is Eulerian. Hence by proposition 3.2, the theorem follows.

Example 3.6: Consider the symmetric group given in example 2.9. The algebraic conservative Petri net for the group S_3, *ACPN (S:S₃)* is shown in Fig.2. Clearly this is a marked graph.

Let $\pi = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$ be the partition of the set of places of S_3, where $\pi_1 = \{p_2, p_3\}$,

$\pi_2 = \{p_5, p_6, p_7\}$, $\pi_3 = \{p_8, p_9\}$, $\pi_4 = \{p_{10}, p_{11}\}$, $\pi_5 = \{p_1, p_4, p_{12}\}$ Such that $^\bullet\pi_1 = \{\alpha, \lambda\}$,

$\pi_1^\bullet = \{\alpha, \lambda\}$. That is , $^\bullet\pi_1 = \pi_1^\bullet$. Therefore $_1$ is both siphon and trap.

Similarly, $^\bullet\pi_2 = \{\beta, \varphi, \lambda\}$, $\pi_2^\bullet = \{\beta, \varphi, \lambda\}$. **That is,** $^\bullet\pi_2 = \pi_2^\bullet$. Therefore $^\bullet\pi_2$ is both siphon and trap. $^\bullet\pi_3 = \{\beta, \sigma\}$, $\pi_3^\bullet = \{\beta, \sigma\}$. That is, $^\bullet\pi_3 = \pi_3^\bullet$. Therefore $_3$ is both siphon and trap. $^\bullet\pi_4 = \{\varphi, \psi\}$, $\pi_4^\bullet = \{\varphi, \psi\}$. **That is,** $^\bullet\pi_4 = \pi_4^\bullet$. Therefore $_4$ is both siphon and trap.

$^\bullet\pi_5 = \{\alpha, \psi, \delta\}$, $\pi_5^\bullet = \{\alpha, \psi, \delta\}$, That is, $^\bullet\pi_5 = \pi_5^\bullet$ Therefore π_5 is both siphon and trap. Hence the underlying directed graph of the marked graph for S_3 is Eulerian.

Again in *ACPN (S:S₃)* there exist a subset $Z = \{p_2, p_6, p_7, p_8, p_{12}, p_4\}$ such that $^\bullet Z = Z^\bullet = T$. Therefore in the underlying directed graph, the edges $\{e_1, e_3, e_4, e_6, e_{10}, e_{11}\}$ corresponding to the places in Z constitutes a directed Hamiltonian circuit.

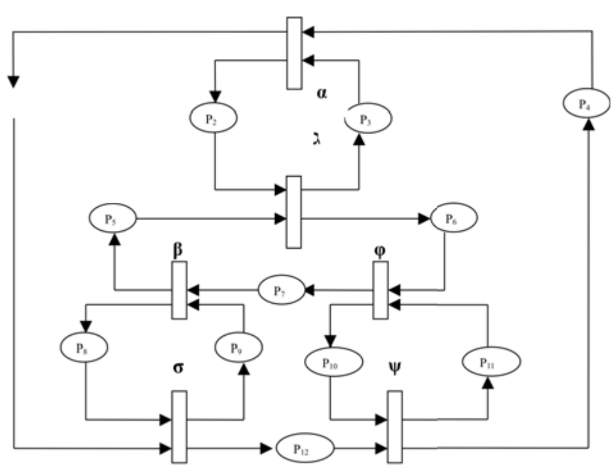

Fig .2 The algebraic conservative Petri net for the group given in example 2.9

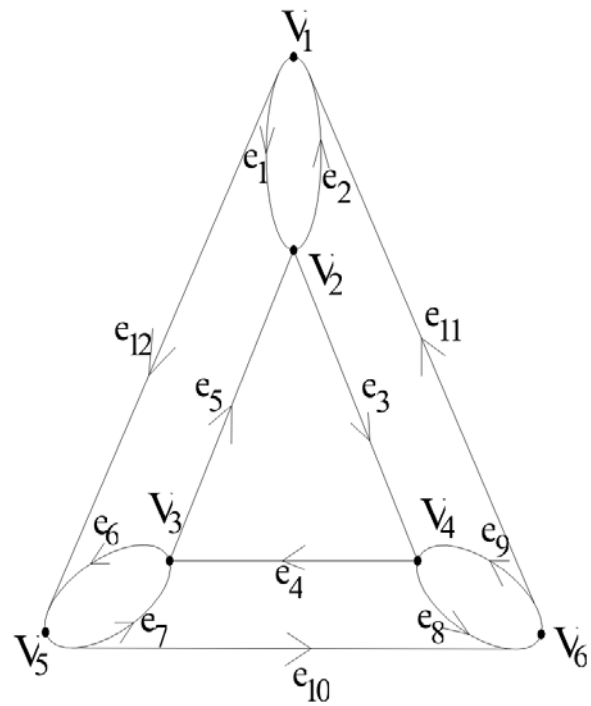

Fig .3 The underlying directed graph for the algebraic conservative Petri net for the Fig .2

Example 3.7: Consider the symmetric group S_5, consisting of 120 elements as follows.

12345	21345	31245	41235	51234
12354	21354	31254	41253	51243
12534	21534	31425	41523	51342
12543	21543	31452	41532	51324
12435	21453	31524	41325	51423
12453	21435	31542	41352	51432
13245	23145	32145	42135	52134
13254	23154	32154	42153	52143
13425	23415	32514	42513	52314
13452	23451	32541	42531	52341
13524	23514	32415	42315	52413
13542	23541	32451	42351	52431
14235	24315	34251	43251	53421
14253	24351	34215	43215	53412
14325	24531	34521	43521	53241
14352	24513	34512	43512	53214
14523	24135	34125	43125	53142
14532	24513	34152	43152	53124
15432	25413	35142	45132	54321
15423	25431	35124	45123	54312
15243	25341	35421	45321	54213
15234	25314	35412	45312	54231
15342	25143	35214	45231	54123
15324	25134	35241	45213	54132

Let $\boldsymbol{S_5}$ be a symmetric group with generating set $S = \{(1\ 2), (1\ 2\ 3\ 4\ 5)\}$. Let $\gamma = (1\ 2)$ and $\Psi = (1\ 2\ 3\ 4\ 5)$. Since S_5 has 120 elements, take the elements of S_5 as the transition set of conservative Petri net. That is, $T = \{t_1, t_2, t_3, \ldots, t_{120}\}$. Based on the theorem, we have 240 places namely $P = \{p_1, p_2, p_3, p_4, \ldots\ldots, p_{240}\}$. The input and outputs of transitions are given in the following table. This leads to the following conservative Petri net *ACPN (S:S₅)*.

$^{\bullet}t_1=\{p_2,p_{125}\}$ $t_1^{\bullet}=\{p_1, p_{121}\}$	$^{\bullet}t_2=\{p_4,p_{130}\}$ $t_2^{\bullet}=\{p_3, p_{126}\}$	$^{\bullet}t_3=\{p_6,p_{135}\}$, $t_3^{\bullet}=\{p_5, p_{131}\}$	$^{\bullet}t_4=\{p_8,p_{140}\}$, $t_4^{\bullet}=\{P_7, p_{136}\}$	$^{\bullet}t_5=\{p_{10},p_{145}\}$, $t_5^{\bullet}=\{p_9, p_{141}\}$
$^{\bullet}t_6=\{p_{12},p_{150}\}$, $t_6^{\bullet}=\{p_{11}, p_{146}\}$	$^{\bullet}t_7=\{p_{14},p_{155}\}$, $t_7^{\bullet}=\{p_{13}, p_{151}\}$	$^{\bullet}t_8=\{p_{16},p_{160}\}$, $t_8^{\bullet}=\{p_{15}, p_{156}\}$	$^{\bullet}t_9=\{p_{18},p_{165}\}$, $t_9^{\bullet}=\{p_{17}, p_{161}\}$	$^{\bullet}t_{10}=\{p_{20},p_{170}\}$, $t_{10}^{\bullet}=\{P_{19}, P_{166}\}$
$^{\bullet}t_{11}=\{p_{22},p_{175}\}$, $t_{11}^{\bullet}=\{p_{21}, p_{171}\}$	$^{\bullet}t_{12}=\{p_{24},p_{180}\}$ $t_{12}^{\bullet}=\{p_{23}, p_{176}\}$	$^{\bullet}t_{13}=\{p_{26},p_{185}\}$ $t_{13}^{\bullet}=\{p_{25}, p_{181}\}$	$^{\bullet}t_{14}=\{p_{28},p_{190}\}$, $t_{14}^{\bullet}=\{p_{27}, p_{186}\}$	$^{\bullet}t_{15}=\{p_{30},p_{195}\}$, $t_{15}^{\bullet}=\{p_{29}, p_{191}\}$
$^{\bullet}t_{16}=\{p_{32},p_{200}\}$, $t_{16}^{\bullet}=\{p_{31}, p_{136}\}$	$^{\bullet}t_{17}=\{p_{34},p_{205}\}$, $t_{17}^{\bullet}=\{p_{33}, p_{201}\}$	$^{\bullet}t_{18}=\{p_{36},p_{210}\}$, $t_{18}^{\bullet}=\{p_{35}, p_{206}\}$	$^{\bullet}t_{19}=\{p_{38},p_{215}\}$ $t_{19}^{\bullet}=\{p_{37}, p_{211}\}$	$^{\bullet}t_{20}=\{p_{40},p_{220}\}$, $t_{20}^{\bullet}=\{p_{39}, p_{216}\}$
$^{\bullet}t_{21}=\{p_{42},p_{225}\}$, $t_{21}^{\bullet}=\{p_{41}, p_{221}\}$	$^{\bullet}t_{22}=\{p_{44},p_{230}\}$, $t_{22}^{\bullet}=\{p_{43}, p_{226}\}$	$^{\bullet}t_{23}=\{p_{46},p_{235}\}$, $t_{23}^{\bullet}=\{p_{45}, p_{231}\}$	$^{\bullet}t_{24}=\{p_{48},p_{240}\}$, $t_{24}^{\bullet}=\{p_{47}, p_{236}\}$	$^{\bullet}t_{25}=\{p_1,p_{226}\}$, $t_{25}^{\bullet}=\{p_2, p_{227}\}$
$^{\bullet}t_{26}=\{p_3,p_{221}\}$, $t_{26}^{\bullet}=\{p_4, p_{222}\}$	$^{\bullet}t_{27}=\{p_5,p_{216}\}$ $t_{27}^{\bullet}=\{p_6, p_{217}\}$	$^{\bullet}t_{28}=\{p_7,p_{211}\}$ $t_{28}^{\bullet}=\{p_8, p_{212}\}$	$^{\bullet}t_{29}=\{p_{11},p_{231}\}$ $t_{29}^{\bullet}=\{p_{12}, p_{232}\}$	$^{\bullet}t_{30}=\{p_9,p_{236}\}$ $t_{30}^{\bullet}=\{p_{10}, p_{237}\}$
$^{\bullet}t_{31}=\{p_{13},p_{131}\}$ $t_{31}^{\bullet}=\{p_{14}, p_{132}\}$	$^{\bullet}t_{32}=\{p_{15},p_{136}\}$ $t_{32}^{\bullet}=\{p_{16}, p_{137}\}$	$^{\bullet}t_{33}=\{p_{17},p_{126}\}$ $t_{33}^{\bullet}=\{p_{18}, p_{127}\}$	$^{\bullet}t_{34}=\{p_{19},p_{121}\}$ $t_{34}^{\bullet}=\{p_{20}, p_{122}\}$	$^{\bullet}t_{35}=\{p_{21},p_{146}\}$ $t_{35}^{\bullet}=\{p_{22}, p_{147}\}$
$^{\bullet}t_{36}=\{p_{23},p_{141}\}$ $t_{36}^{\bullet}=\{p_{24}, p_{142}\}$	$^{\bullet}t_{37}=\{p_{29},p_{156}\}$ $t_{37}^{\bullet}=\{p_{30}, p_{157}\}$	$^{\bullet}t_{38}=\{p_{31},p_{151}\}$ $t_{38}^{\bullet}=\{p_{32}, p_{152}\}$	$^{\bullet}t_{39}=\{p_{35},p_{161}\}$ $t_{39}^{\bullet}=\{p_{36}, p_{162}\}$	$^{\bullet}t_{40}=\{p_{33},p_{166}\}$ $t_{40}^{\bullet}=\{p_{34}, p_{167}\}$
$^{\bullet}t_{41}=\{p_{25},p_{171}\}$ $t_{41}^{\bullet}=\{p_{26}, p_{172}\}$	$^{\bullet}t_{42}=\{p_{27},p_{176}\}$ $t_{42}^{\bullet}=\{p_{28}, p_{177}\}$	$^{\bullet}t_{43}=\{p_{39},p_{196}\}$ $t_{43}^{\bullet}=\{p_{40}, p_{197}\}$	$^{\bullet}t_{44}=\{p_{37},p_{191}\}$ $t_{44}^{\bullet}=\{p_{38}, p_{192}\}$	$^{\bullet}t_{45}=\{p_{45},p_{181}\}$ $t_{45}^{\bullet}=\{p_{46}, p_{182}\}$
$^{\bullet}t_{46}=\{p_{47},p_{186}\}$ $t_{46}^{\bullet}=\{p_{48}, p_{187}\}$	$^{\bullet}t_{47}=\{p_{41},p_{206}\}$ $t_{47}^{\bullet}=\{p_{42}, p_{207}\}$	$^{\bullet}t_{48}=\{p_{43},p_{201}\}$ $t_{48}^{\bullet}=\{p_{44}, p_{202}\}$	$^{\bullet}t_{49}=\{p_{50},p_{202}\}$ $t_{49}^{\bullet}=\{p_{49}, p_{203}\}$	$^{\bullet}t_{50}=\{p_{52},p_{207}\}$ $t_{50}^{\bullet}=\{p_{51}, p_{208}\}$
$t_{51}=\{p_{54},p_{187}\}$ $t_{51}^{\bullet}=\{p_{53}, p_{188}\}$	$^{\bullet}t_{52}=\{p_{56},p_{182}\}$ $t_{52}^{\bullet}=\{p_{55}, p_{183}\}$	$^{\bullet}t_{53}=\{p_{58},p_{197}\}$ $t_{53}^{\bullet}=\{p_{57}, p_{198}\}$	$^{\bullet}t_{54}=\{p_{60},p_{192}\}$ $t_{54}^{\bullet}=\{p_{59}, p_{193}\}$	$^{\bullet}t_{55}=\{p_{49},p_{217}\}$ $t_{55}^{\bullet}=\{p_{50}, p_{218}\}$
$^{\bullet}t_{56}=\{p_{51},p_{212}\}$ $t_{56}^{\bullet}=\{p_{52}, p_{213}\}$	$^{\bullet}t_{57}=\{p_{57},p_{232}\}$ $t_{57}^{\bullet}=\{p_{58}, p_{233}\}$	$^{\bullet}t_{58}=\{p_{59},p_{237}\}$ $t_{58}^{\bullet}=\{p_{60}, p_{238}\}$	$^{\bullet}t_{59}=\{p_{53},p_{222}\}$ $t_{59}^{\bullet}=\{p_{54}, p_{223}\}$	$^{\bullet}t_{60}=\{p_{55},p_{227}\}$ $t_{60}^{\bullet}=\{p_{56}, p_{228}\}$
$^{\bullet}t_{61}=\{p_{62},p_{132}\}$ $t_{61}^{\bullet}=\{p_{61}, p_{133}\}$	$^{\bullet}t_{62}=\{p_{64},p_{137}\}$ $t_{62}^{\bullet}=\{p_{63}, p_{138}\}$	$^{\bullet}t_{63}=\{p_{66},p_{127}\}$ $t_{63}^{\bullet}=\{p_{65}, p_{128}\}$	$^{\bullet}t_{64}=\{p_{66},p_{122}\}$ $t_{64}^{\bullet}=\{p_{65}, p_{123}\}$	$^{\bullet}t_{65}=\{p_{63},p_{147}\}$ $t_{65}^{\bullet}=\{p_{64}, p_{148}\}$
$^{\bullet}t_{66}=\{p_{61},p_{142}\}$ $t_{66}^{\bullet}=\{p_{62}, p_{143}\}$	$^{\bullet}t_{67}=\{p_{68},p_{162}\}$ $t_{67}^{\bullet}=\{p_{67}, p_{163}\}$	$^{\bullet}t_{68}=\{p_{70},p_{167}\}$ $t_{68}^{\bullet}=\{p_{69}, p_{168}\}$	$^{\bullet}t_{69}=\{p_{72},p_{157}\}$ $t_{69}^{\bullet}=\{p_{71}, p_{158}\}$	$^{\bullet}t_{70}=\{p_{71},p_{152}\}$ $t_{70}^{\bullet}=\{p_{72}, p_{153}\}$
$^{\bullet}t_{71}=\{p_{69},p_{177}\}$ $t_{71}^{\bullet}=\{p_{70}, p_{178}\}$	$^{\bullet}t_{72}=\{p_{67},p_{172}\}$ $t_{72}^{\bullet}=\{p_{68}, p_{173}\}$	$^{\bullet}t_{73}=\{p_{74},p_{168}\}$ $t_{73}^{\bullet}=\{p_{73}, p_{169}\}$	$^{\bullet}t_{74}=\{p_{76},p_{163}\}$ $t_{74}^{\bullet}=\{p_{75}, p_{164}\}$	$^{\bullet}t_{75}=\{p_{78},p_{153}\}$ $t_{75}^{\bullet}=\{p_{77}, p_{154}\}$
$^{\bullet}t_{76}=\{p_{80},p_{158}\}$ $t_{76}^{\bullet}=\{p_{79}, p_{159}\}$	$^{\bullet}t_{77}=\{p_{82},p_{178}\}$ $t_{77}^{\bullet}=\{p_{81}, p_{179}\}$	$^{\bullet}t_{78}=\{p_{84},p_{173}\}$ $t_{78}^{\bullet}=\{p_{83}, p_{174}\}$	$^{\bullet}t_{79}=\{p_{73},p_{198}\}$ $t_{79}^{\bullet}=\{p_{74}, p_{199}\}$	$^{\bullet}t_{80}=\{p_{75},p_{193}\}$ $t_{80}^{\bullet}=\{p_{76}, p_{194}\}$
$^{\bullet}t_{81}=\{p_{77},p_{183}\}$ $t_{81}^{\bullet}=\{p_{78}, p_{184}\}$	$^{\bullet}t_{82}=\{p_{79},p_{188}\}$ $t_{82}^{\bullet}=\{p_{80}, p_{189}\}$	$^{\bullet}t_{83}=\{p_{81},p_{208}\}$ $t_{83}^{\bullet}=\{p_{82}, p_{209}\}$	$^{\bullet}t_{84}=\{p_{83},p_{203}\}$ $t_{84}^{\bullet}=\{p_{84}, p_{204}\}$	$^{\bullet}t_{85}=\{p_{86},p_{218}\}$ $t_{85}^{\bullet}=\{p_{85}, p_{219}\}$
$^{\bullet}t_{86}=\{p_{88},p_{213}\}$ $t_{86}^{\bullet}=\{p_{87}, p_{214}\}$	$^{\bullet}t_{87}=\{p_{90},p_{223}\}$ $t_{87}^{\bullet}=\{p_{89}, p_{224}\}$	$^{\bullet}t_{88}=\{p_{89},p_{228}\}$ $t_{88}^{\bullet}=\{p_{90}, p_{229}\}$	$^{\bullet}t_{89}=\{p_{87},p_{233}\}$ $t_{89}^{\bullet}=\{p_{88}, p_{234}\}$	$^{\bullet}t_{90}=\{p_{85},p_{238}\}$ $t_{90}^{\bullet}=\{p_{86}, p_{239}\}$
$^{\bullet}t_{91}=\{p_{92},p_{128}\}$ $t_{91}^{\bullet}=\{p_{91}, p_{129}\}$	$^{\bullet}t_{92}=\{p_{94},p_{123}\}$ $t_{92}^{\bullet}=\{p_{93}, p_{124}\}$	$^{\bullet}t_{93}=\{p_{96},p_{138}\}$ $t_{93}^{\bullet}=\{p_{95}, p_{139}\}$	$^{\bullet}t_{94}=\{p_{95},p_{133}\}$ $t_{94}^{\bullet}=\{p_{96}, p_{134}\}$	$^{\bullet}t_{95}=\{p_{91},p_{148}\}$ $t_{95}^{\bullet}=\{p_{92}, p_{149}\}$
$^{\bullet}t_{96}=\{p_{93},p_{143}\}$ $t_{96}^{\bullet}=\{p_{94}, p_{144}\}$	$^{\bullet}t_{97}=\{p_{98},p_{124}\}$ $t_{97}^{\bullet}=\{p_{97}, p_{125}\}$	$^{\bullet}t_{98}=\{p_{100},p_{129}\}$ $t_{98}^{\bullet}=\{p_{99}, p_{130}\}$	$^{\bullet}t_{99}=\{p_{102},p_{149}\}$ $t_{99}^{\bullet}=\{p_{101}, p_{150}\}$	$^{\bullet}t_{100}=\{p_{104},p_{144}\}$ $t_{100}^{\bullet}=\{p_{103}, p_{145}\}$
$^{\bullet}t_{101}=\{p_{106},p_{134}\}$ $t_{101}^{\bullet}=\{p_{105}, p_{135}\}$	$^{\bullet}t_{102}=\{p_{108},p_{139}\}$ $t_{102}^{\bullet}=\{p_{107}, p_{140}\}$	$^{\bullet}t_{103}=\{p_{97},p_{134}\}$ $t_{103}^{\bullet}=\{p_{98}, p_{155}\}$	$^{\bullet}t_{104}=\{p_{99},p_{159}\}$ $t_{104}^{\bullet}=\{p_{100}, p_{160}\}$	$^{\bullet}t_{105}=\{p_{103},p_{165}\}$ $t_{105}^{\bullet}=\{p_{104}, p_{165}\}$
$^{\bullet}t_{106}=\{p_{101},p_{169}\}$ $t_{106}^{\bullet}=\{p_{102}, p_{170}\}$	$^{\bullet}t_{107}=\{p_{105},p_{174}\}$ $t_{107}^{\bullet}=\{p_{106}, p_{175}\}$	$^{\bullet}t_{108}=\{p_{107},p_{179}\}$ $t_{108}^{\bullet}=\{p_{108}, p_{180}\}$	$^{\bullet}t_{109}=\{p_{110},p_{209}\}$ $t_{109}^{\bullet}=\{p_{109}, p_{210}\}$	$^{\bullet}t_{110}=\{p_{109},p_{204}\}$ $t_{110}^{\bullet}=\{p_{110}, p_{205}\}$
$^{\bullet}t_{111}=\{p_{112},p_{199}\}$ $t_{111}^{\bullet}=\{p_{111}, p_{200}\}$	$^{\bullet}t_{112}=\{p_{114},p_{194}\}$ $t_{112}^{\bullet}=\{p_{113}, p_{195}\}$	$^{\bullet}t_{113}=\{p_{111},p_{189}\}$ $t_{113}^{\bullet}=\{p_{112}, p_{190}\}$	$^{\bullet}t_{114}=\{p_{113},p_{184}\}$ $t_{114}^{\bullet}=\{p_{114}, p_{185}\}$	$^{\bullet}t_{115}=\{p_{116},p_{214}\}$ $t_{115}^{\bullet}=\{p_{115}, p_{215}\}$
$^{\bullet}t_{116}=\{p_{115},p_{219}\}$ $t_{116}^{\bullet}=\{p_{116}, p_{220}\}$	$^{\bullet}t_{117}=\{p_{118},p_{239}\}$ $t_{117}^{\bullet}=\{p_{117}, p_{240}\}$	$^{\bullet}t_{118}=\{p_{120},p_{234}\}$ $t_{118}^{\bullet}=\{p_{119}, p_{235}$	$^{\bullet}t_{119}=\{p_{117},p_{229}\}$ $t_{119}^{\bullet}=\{p_{118}, p_{230}\}$	$^{\bullet}t_{120}=\{p_{119},p_{224}\}$ $t_{120}^{\bullet}=\{p_{120}, p_{225}\}$

Let $\pi = \{\pi_1, \pi_2, \pi_3, \ldots, \pi_{84}\}$ be the partition of the set of places of S_5. where

$\pi_1 = \{p_{121}, p_{122}, p_{123}, p_{124}, p_{125}\}$, $\pi_2 = \{p_{126}, p_{127}, p_{128}, p_{129}, p_{130}\}$, $\pi_3 = \{p_{131}, p_{132}, p_{133}, p_{134}, p_{135}\}$,
$\pi_4 = \{p_{136}, p_{137}, p_{138}, p_{139}, p_{140}\}$, $\pi_5 = \{p_{141}, p_{142}, p_{143}, p_{144}, p_{145}\}$, $\pi_6 = \{p_{146}, p_{147}, p_{148}, p_{149}, p_{150}\}$,
$\pi_7 = \{p_{151}, p_{152}, p_{153}, p_{154}, p_{155}\}$, $\pi_8 = \{p_{156}, p_{157}, p_{158}, p_{159}, p_{160}\}$, $\pi_9 = \{p_{161}, p_{162}, p_{163}, p_{164}, p_{165}\}$,
$\pi_{10} = \{p_{166}, p_{167}, p_{168}, p_{169}, p_{170}\}$, $\pi_{11} = \{p_{171}, p_{172}, p_{173}, p_{174}, p_{175}\}$, $\pi_{12} = \{p_{176}, p_{177}, p_{178}, p_{179}, p_{180}\}$,
$\pi_{13} = \{p_{181}, p_{182}, p_{183}, p_{184}, p_{185}\}$, $\pi_{14} = \{p_{186}, p_{187}, p_{188}, p_{189}, p_{190}\}$, $\pi_{15} = \{p_{191}, p_{192}, p_{193}, p_{194}, p_{195}\}$,
$\pi_{16} = \{p_{196}, p_{197}, p_{198}, p_{199}, p_{200}\}$, $\pi_{17} = \{p_{201}, p_{202}, p_{203}, p_{204}, p_{205}\}$, $\pi_{18} = \{p_{206}, p_{207}, p_{208}, p_{209}, p_{210}\}$,
$\pi_{19} = \{p_{211}, p_{212}, p_{213}, p_{214}, p_{215}\}$, $\pi_{20} = \{p_{216}, p_{217}, p_{218}, p_{219}, p_{220}\}$, $\pi_{21} = \{p_{221}, p_{222}, p_{223}, p_{224}, p_{225}\}$,
$\pi_{22} = \{p_{226}, p_{227}, p_{228}, p_{229}, p_{230}\}$, $\pi_{23} = \{p_{231}, p_{232}, p_{233}, p_{234}, p_{235}\}$, $\pi_{24} = \{p_{236}, p_{237}, p_{238}, p_{239}, p_{240}\}$,
$\pi_{25} = \{p_1, p_2\}$, $\pi_{26} = \{p_3, p_4\}$, $\pi_{27} = \{p_5, p_6\}$, $\pi_{28} = \{p_7, p_8\}$, $\pi_{29} = \{p_9, p_{10}\}$, $\pi_{30} = \{p_{11}, p_{12}\}$,
$\pi_{31} = \{p_{13}, p_{14}\}$, $\pi_{32} = \{p_{15}, p_{16}\}$, $\pi_{33} = \{p_{17}, p_{18}\}$, $\pi_{34} = \{p_{19}, p_{20}\}$, $\pi_{35} = \{p_{21}, p_{22}\}$, $\pi_{36} = \{p_{23}, p_{24}\}$,
$\pi_{37} = \{p_{25}, p_{26}\}$, $\pi_{38} = \{p_{27}, p_{28}\}$, $\pi_{39} = \{p_{29}, p_{30}\}$, $\pi_{40} = \{p_{31}, p_{32}\}$, $\pi_{41} = \{p_{33}, p_{34}\}$, $\pi_{42} = \{p_{35}, p_{36}\}$,
$\pi_{43} = \{p_{37}, p_{38}\}$, $\pi_{44} = \{p_{39}, p_{40}\}$, $\pi_{45} = \{p_{41}, p_{42}\}$, $\pi_{46} = \{p_{43}, p_{44}\}$, $\pi_{47} = \{p_{45}, p_{46}\}$, $\pi_{48} = \{p_{47}, p_{48}\}$,
$\pi_{49} = \{p_{49}, p_{50}\}$, $\pi_{50} = \{p_{51}, p_{52}\}$, $\pi_{51} = \{p_{53}, p_{54}\}$, $\pi_{52} = \{p_{55}, p_{56}\}$, $\pi_{53} = \{p_{57}, p_{58}\}$, $\pi_{54} = \{p_{59}, p_{60}\}$,
$\pi_{55} = \{p_{61}, p_{62}\}$, $\pi_{56} = \{p_{63}, p_{64}\}$, $\pi_{57} = \{p_{65}, p_{66}\}$, $\pi_{58} = \{p_{67}, p_{68}\}$, $\pi_{59} = \{p_{69}, p_{70}\}$, $\pi_{60} = \{p_{71}, p_{72}\}$,
$\pi_{61} = \{p_{73}, p_{74}\}$, $\pi_{62} = \{p_{75}, p_{76}\}$, $\pi_{63} = \{p_{77}, p_{78}\}$, $\pi_{64} = \{p_{79}, p_{80}\}$, $\pi_{65} = \{p_{81}, p_{82}\}$, $\pi_{66} = \{p_{83}, p_{84}\}$,
$\pi_{67} = \{p_{85}, p_{86}\}$, $\pi_{68} = \{p_{87}, p_{88}\}$, $\pi_{69} = \{p_{89}, p_{90}\}$, $\pi_{70} = \{p_{91}, p_{92}\}$, $\pi_{71} = \{p_{93}, p_{94}\}$, $\pi_{72} = \{p_{95}, p_{96}\}$,
$\pi_{73} = \{p_{97}, p_{98}\}$, $\pi_{74} = \{p_{99}, p_{100}\}$, $\pi_{75} = \{p_{101}, p_{102}\}$, $\pi_{76} = \{p_{103}, p_{104}\}$, $\pi_{77} = \{p_{105}, p_{106}\}$,
$\pi_{78} = \{p_{107}, p_{108}\}$, $\pi_{79} = \{p_{109}, p_{110}\}$, $\pi_{80} = \{p_{111}, p_{112}\}$, $\pi_{81} = \{p_{113}, p_{114}\}$, $\pi_{82} = \{p_{115}, p_{116}\}$,
$\pi_{83} = \{p_{117}, p_{118}\}$, $\pi_{84} = \{p_{119}, p_{120}\}$

The above *ACPN ($S:S_5$)* is clearly a marked graph. This *ACPN ($S:S_5$)* has a subset of places which are both siphon and trap such that the input transitions equal the output transitions and both of them equal to the set of all transitions of the marked graph and hence that the underlying directed graph of this marked graph is Hamiltonian. Since there exists a partition for a place set P such that each block in the partition of the set of places of the marked graph is both siphon and trap, the underlying directed graph for *ACPN ($S:S_5$)* is Eulerian.

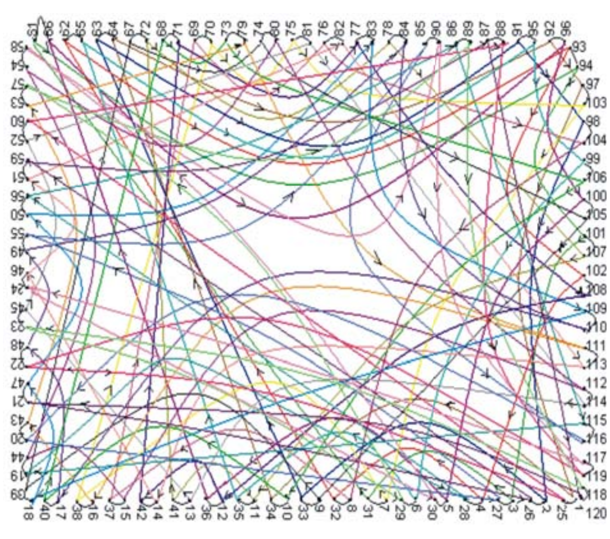

Fig .4 The underlying directed graph for the algebraic conservative Petri net given in example 3.6

IV. CONCLUSION

In this paper we introduced a new sub class of Petri nets called algebraic conservative Petri nets (ACPN) for a given symmetric group S_n. We proved that the resulting Petri net (ACPN) is a marked graph . In particular, for the groups S_3 and S_5 , we show that each of the ACPN associated with these groups has a subset of places which are both siphon and trap such that the input transitions equal the output transitions and both of them equal to the set of all transitions of these algebraic conservative Petri nets and hence that the underlying directed graphs of these algebraic conservative Petri nets are **Hamiltonian.** Also we shown that the algebraic conservative Petri nets associated with S_3 and S_5 has decompositions

$\pi = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$ and $\pi = \{\pi_1, \pi_2, \pi_3, \pi_4, \ldots, \pi_{84}\}$ respectively, for the sets of places such that each block π_i is both siphon and trap and hence the underlying directed graphs of these algebraic conservative Petri nets associated with S_3 and S_5 are Eulerian.

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