AN EFFICIENT REPRESENTATION OF CHARACTERIZATION OF SUPER STRONGLY PERFECT GRAPHS IN SOME INTER CONNECTION NETWORKS

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Abstract

A Graph G is Super Strongly Perfect Graph if every induced sub graph H of G possesses a minimal dominating set that meet all the maximal complete sub graphs of H. In this paper we have analyzed the structure of super strongly perfect graphs in some inter connection networks like Butterfly, Wrapped Butterfly and Benes Networks. We have given the characterization of Super Strongly Perfect graphs in Butterfly, Wrapped Butterfly and Benes Networks. Also we have investigated the relationship between diameter, domination and co - domination numbers of Butterfly, Wrapped Butterfly and Benes Networks.

Key words: Super Strongly Perfect Graph, Minimal Dominating Set, Butterfly, Wrapped Butterfly and Benes Networks.

I. INTRODUCTION

The field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. Also Graph theoretical ideas are highly utilized by computer science applications. Especially in research areas of computer science such data mining, image segmentation, clustering, image capturing, networking etc., Similarly modeling of network topologies can be done using graph concepts. In the same way the most important concept of graph colouring is utilized in resource allocation, scheduling. This leads to the development of new algorithms and new theorems that can be used in tremendous applications (8). Since the emergence of parallel processing in the 1960s, numerous networks have been proposed for connecting the processing nodes in distributed multicomputers, to the extent that a "sea of interconnection networks" is said to exist (7). An implication of this terminology is that new networks, or designers trying to make sense of the wide array of options available to them, might drown in this sea. It is for these reasons that classes of networks offering cost - performance tradeoffs within a wide range are extremely useful, because membership in the same class allows the application of theoretical results to make the task of performance evaluation both tractable and meaningful.

The fact that Butterfly and Benes Networks are excellent models for interconnection networks, investigated in connection with parallel processing and distributed computation, is widely acknowledged (4). Butterfly, Wrapped Butterfly and Benes Networks have been described in the technical literature and still others await discovery and these Networks also play an important role in studies relating the three network parameters of size, node degree, and diameter. Collective communication operations frequently occur in parallel computing, and their performance often determines the overall running time of an application. Butterfly Network is a popular interconnection network used in parallel computing (5). It is also used in peer - to - peer networks (4). Buttery Network supports mappings of many signal processing algorithms such as the fast Fourier transform as well as many basic structures such as cycles and trees.

II. BASIC CONCEPTS

In this paper, graphs are finite, simple, that is, they have no loops or multiple edges and undirected. Let G be a graph. A *clique / Maximal complete sub graph* in G is a set $X \subseteq V(G)$ of pair wise adjacent vertices. A subset D of $V-(G)$ is called a *dominating set* if every vertex in $V-D$ is adjacent to at least one vertex in D. A subset S of V is said to be a *minimal dominating set* if $S-\{u\}$ is not a dominating set for any $u \in S$. The domination number $\gamma(G)$ of G is the smallest size of a dominating set of G. The domination number of its complement $\overline{G}$ is called the co-domination number of $\overline{G}$ and is denoted by $\gamma(\overline{G})$ or simply. $\overline{V}$ A shortest $u-v$ path of a connected graph G is often called a geodesic. The diameter of G is the length of any longest geodesic and it is denoted by diam (G).

III. OUR RESULTS IN SUPER STRONGLY PERFECT GRAPH

The most popular bounded - degree derivative network of the hypercube is the butterfly network. The Wrapped Butterfly Network is the butterfly Network with wrap around connection between some nodes and the Benes Network consists of back - to - back butterflies. There exist a number of topological representations that are used to describe butterfly - like architectures.

In this paper, we have identified a new topological representation of Butterfly, Wrapped Butterfly and Benes Networks. We have investigated the structure of Super Strongly Perfect Graph in Butterfly, Wrapped Butterfly and Benes Networks. We have presented the characterization of Super Strongly Perfect graphs in Butterfly, Wrapped Butterfly and Benes Networks. Also we have analyzed the relationship between diameter, domination and co - domination numbers of Butterfly, Wrapped Butterfly and Benes Networks.

A. *Super strongly perfect graph*

A Graph $G=(V, E)$ is Super Strongly Perfect if every induced sub graph H of G possesses a minimal dominating set that meet all the maximal complete sub graphs of H.

Example 1

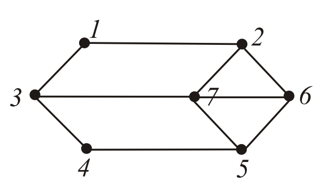

Fig. 1. Super Strongly Perfect Graph

Here, {3, 6} is a minimal dominating set which meet all maximal cliques of G.

Example 2

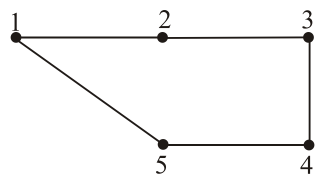

Fig. 2. Non - Super Strongly Perfect Graph

Here, {1, 3} is a minimal dominating set which does not meet all maximal cliques of G.

IV. CYCLE GRAPH

A Cycle graph or Circular graph is a graph that consists of a single cycle or in other words some number of vertices connected in a closed chain and it is denoted by C_n. The number of vertices in C_n equals the number of edges. The cycle graph with even number of vertices is called an even cycle and the cycle graph with odd number of vertices is called an odd cycle.

A. *Theorem (2)*

Let $G=(V, E)$ be a graph with number of vertices n, where $n \geq 5$. Then G is Super Strongly Perfect if and only if it does not contain an odd cycle of length at least 5 as an induced sub graph.

V. BIPARTITE GRAPH

A bipartite graph (or bigraph) is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V. That is no two graph vertices within U or V are adjacent. Hence U and V are independent sets. A complete bipartite graph is a bipartite graph such that every pair of graph vertices in the two sets are adjacent.

Example 3

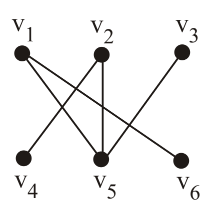

Fig. 3. Bipartite graph

Here, $\{v_1, v_2, v_5\}$ is a minimal dominating set which meet all maximal cliques of G.

A. *Theorem (2)*

Every bipartite graph is Super Strongly Perfect.

B. *Theorem (2)*

Let G be graph with maximal complete sub graph K_2. Then G is bipartite if and only if it is Super Strongly Perfect.

C. Theorem (2)

Let G be a complete bipartite graph which is Super Strongly Perfect, then $\gamma(G) = 2$ if and only if $\gamma(\overline{G}) = 2$.

D. Theorem (2)

Let G be a complete bipartite graph which is Super Strongly Perfect, then diam $(G) = 2$ if and only if diam $(\overline{G})$ is not defined.

E. Theorem (2)

Let G be a bipartite graph with no isolated vertex which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

VI. BUTTERFLY NETWORK

Many interconnection networks have been proposed as suitable topologies for parallel computers. Among them, Butterfly networks have received particular attention, due to their interesting structure. First, we have to warn the reader that under the name Butterfly and with the same notation, different networks are described in the literature. Indeed, while some authors consider the Butterfly networks to be multistage networks used to route permutations, others consider them to be point - to -point networks. In what follows, we will use the term Butterfly for the multistage version and we will use Leighton's terminology (5), namely wrapped Butterfly, for the point - to - point version. Furthermore, these networks can be considered either as undirected or directed. We represent networks as undirected graphs whose nodes represent processors and whose edges represent interprocessor communication links.

The set V of nodes of an $r-$ dimensional Butterfly correspond to pairs $[w, i]$, where i is the dimension or level of a node $(0 \leq i \leq r)$ and w is an r-bit binary number that denotes the row of the node. Two nodes $w, i >$ and $< w', i' >$ are linked by an edge if and only if $i' = i + 1$ and either

1. w and w' are identical, or

2. w and w' differ in precisely the i^{th} bit.

The edges in the network are undirected. An r-dimensional butterfly is denoted by BF (r). The r-dimensional butterfly has $(r+1)2^r$ nodes and $r2^{r+1}$ edges (1). Every Butterfly network is Bipartite.

Example 4

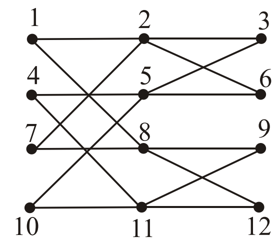

Fig. 4. A 2 - dimensional Butterfly Network

A. Theorem

Every Butterfly Network is Super Strongly Perfect.

Proof:

Let G be a Butterfly Network.

$\Rightarrow G$ does not contain an odd cycle as an induced sub graph.

Now, by the theorem 4.1, G is Super Strongly Perfect.

Hence every Butterfly Network is Super Strongly Perfect.

B. Theorem

Let G be a 1 - dimensional Butterfly Network which is Super Strongly Perfect, then $\gamma(G) = 2$ if and only if $\gamma(\overline{G}) = 2$.

Proof:

Let G be a 1 - dimensional Butterfly Network which is Super Strongly Perfect

Since G is a complete bipartite graph, this theorem is proved by the theorem 5.3.

C. Proposition

Let G be an r-dimensional Butterfly Network, $r \geq 2$, which is Super Strongly Perfect, then $(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

D. Observation

Let G be an r - dimensional Butterfly Network which is Super Strongly Perfect, then $\gamma(G) = \lfloor (r+1)/2 \lfloor 2^r$.

E. Theorem

Let G be a 1 - dimensional Butterfly Network which is Super Strongly Perfect, then diam $(G) = 2$ if and only if diam $(\overline{G})$ is not defined.

Proof:

Let G be a 1 - dimensional Butterfly Network which is Super Strongly Perfect

Since G is a complete bipartite graph, this theorem is proved by the theorem 5.4.

F. Proposition

Let G be an r - dimensional Butterfly Network, $r \geq 2$, which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if diam $(\overline{G}) < 3$.

G. Theorem

Let G be an r - dimensional Butterfly Network, $r \geq 2$, which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

Proof:65

Let G be a Butterfly Network which is Super Strongly Perfect.

Since G is a bipartite graph, this theorem is proved by the theorem 5.5.

VII. WRAPPED BUTTERFLY NETWORK

Improving the communication characteristics of a parallel machine is a challenging problem because of the many conflicting demands on the interconnection networks. The wrapped butterfly network represents a good trade - off between the cost and the performance of a parallel machine. It has a large number of processors, fixed node degree, symmetry, and ability to support a variety of parallel algorithms (6). In BF (r) when the nodes of level 0 are merged with those in level r a new structure called the wrapped butterfly is obtained (5). The r-dimensional wrapped butterfly has $r2^r$ nodes, each of degree 4 and $r2^{r+1}$ edges. We consider the class of wrapped butterflies $WB(r)$, where r is a positive integer. Having this easily described interconnection, $WB(r)$ is found to be vertex symmetric and possess many other interesting structure properties. In fact, $WB(r)$ has received much attention as a good model in network design and it is universal in the sense that it can efficiently simulate an arbitrary bounded - degree network (6).

Example 5

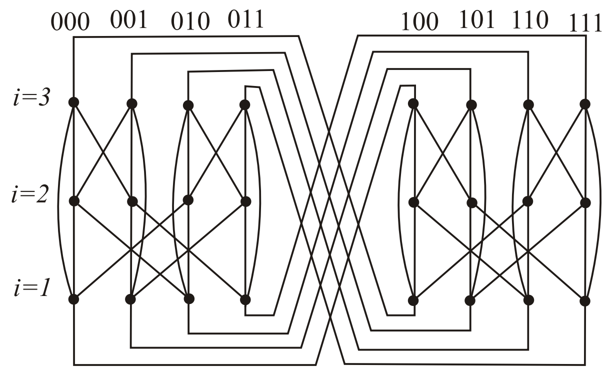

Fig. 5. A 3 - dimensional Wrapped Butterfly Network

A. Theorem

Let G be an r-dimensional Wrapped Butterfly Network, where $r \geq 2$. If r is even, then G is Super Strongly Perfect.

Proof:

Let G be an r-dimensional Wrapped Butterfly Network, where $r \geq 2$, and r is even.

$\Rightarrow G$ does not contain an odd cycle as an induced sub graph.

Now, by the theorem 4.1, G is Super Strongly Perfect.

Hence every r-dimensional Wrapped Butterfly Network, where $r \geq 2$ and r is even, is Super Strongly Perfect.

B. Theorem

Let G be an r-dimensional Wrapped Butterfly Network, where $r = 3$, then G is Super Strongly Perfect.

Proof:

Let G be an r-dimensional Wrapped Butterfly Network, where $r = 3$.

$\Rightarrow G$ does not contain an odd cycle as an induced sub graph.

Now, by the theorem 4.1, G is Super Strongly Perfect.

Hence every r-dimensional Wrapped Butterfly Network, where $r = 3$, is Super Strongly Perfect.

C. Theorem

Let G be an r-dimensional Wrapped Butterfly Network, where $r > 3$. If r is odd, then G is Non - Super Strongly Perfect.

Proof:

Let G be an r-dimensional Wrapped Butterfly Network, where $r > 3$, and r is odd.

$\Rightarrow G$ contains an odd cycle as an induced sub graph.

Now, by the theorem 4.1, G is Non - Super Strongly Perfect.

Hence every r - dimensional Wrapped Butterfly Network, where $r > 3$ and r is odd, is Non - Super Strongly Perfect.

D. Theorem

Let G be a 1 - dimensional Wrapped Butterfly Network which is Super Strongly Perfect, then $\gamma(G) = 2$ if and only if $\gamma(\overline{G}) = 2$.

Proof:

Let G be a 1 - dimensional Wrapped Butterfly Network which is Super Strongly Perfect.

Since G is a complete bipartite graph, this theorem is proved by the theorem 5.3.

E. Proposition

Let G be an r - dimensional Wrapped Butterfly Network, $r \geq 2$ and r is even, which is Super Strongly Perfect, then $\gamma(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

F. Observation

Let G be an r - dimensional Wrapped Butterfly Network, $r \geq 2$ and r is even, which is Super Strongly Perfect, then $\gamma(G) = \left[\frac{r}{2}\right] 2^r$.

G. Theorem

Let G be a 1 - dimensional Wrapped Butterfly Network which is Super Strongly Perfect, then diam $(G) = 2$ if and only if diam $(\overline{G})$ is not defined.

Proof:

Let G be a 1 - dimensional Wrapped Butterfly Network which is Super Strongly Perfect.

Since G is a complete bipartite graph, this theorem is proved by the theorem 5.4.

H. Proposition

Let G be an r - dimensional Wrapped Butterfly Network, $r \geq 2$ and r is even, which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if diam $(\overline{G}) \leq 3$.

I. Theorem

Let G be an r - dimensional Wrapped Butterfly Network, $r \geq 2$ and r is even, which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

Proof:

Let G be an r - dimensional Wrapped Butterfly Network, $r \geq 2$ and r is even, which is Super Strongly Perfect.

Since G is a bipartite graph, this theorem is proved by the theorem 5.5.

VIII. BENES NETWORK

An r - dimensional Benes network has $2r + 1$ levels, each level with $2r$ nodes. The level zero to level r vertices in the network forms an r - dimensional butterfly. The middle level of the Benes network is shared by these butterflies. As butterfly is known for FFT, Benes is known for permutation routing (normal network). An r - dimensional Benes network is denoted by $B(r)$. Even though the Benes network consists of back - to - back butterflies, there is a subtle structural difference between Benes and butterfly. The removal of level 0 nodes of $BF(r)$ leaves two disjoint copies of $BF(r-1)$. In the same way, the removal of level r nodes of $BF(r)$ leaves two disjoint copies of $BF(r-1)$. This recursive structure can be viewed in another way. The removal of level 0 nodes and level r nodes (nodes of degree 2) of $BF(r)$ leaves 4 disjoint copies of a $BF(r-2)$. However the removal of level 0 nodes and level $2r$ nodes (nodes of degree 2) of $B(r)$ leaves 2 disjoint copies of a $B(r-1)$. In other words, the butterfly has dual symmetry in which the

Benes does not have the dual symmetry property (1). Every Benes network is Bipartite.

Example 6

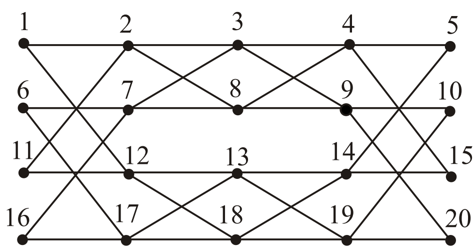

Fig. 6. A 2 - dimensional Benes Network

A. Theorem

Every Benes Network is Super Strongly Perfect.

Proof:

Let G be a Benes Network.

Z G does not contain an odd cycle as an induced sub graph.

Now, by the theorem 4.1, G is Super Strongly Perfect.

Hence every Benes Network is Super Strongly Perfect.

B. Theorem

Let G be a 1 - dimensional Benes Network which is Super Strongly Perfect, then $\gamma(G) = 2$ if and only if $\gamma(\overline{G}) = 2$.

Proof:

Let G be a 1 - dimensional Benes Network which is Super Strongly Perfect

Since G is a complete bipartite graph, this theorem is proved by the theorem 5.3.

C. Proposition

Let G be an r - dimensional Benes Network, $r \geq 2$, which is Super Strongly Perfect, then $\gamma(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

D. Observation

Let G be an r - dimensional Benes Network which is Super Strongly Perfect, then $\gamma(G) = \lfloor (2r+1)/2 \rfloor 2^r$.

E. Theorem

Let G be a 1 - dimensional Benes Network which is Super Strongly Perfect, then diam $(G) = 2$ if and only if diam $(\overline{G})$ is not defined.

Proof:

Let G be a 1 - dimensional Benes Network which is Super Strongly Perfect.

Since G is a complete bipartite graph, this theorem is proved by the theorem 5.4.

F. Proposition

Let G be an r - dimensional Benes Network, $r \geq 2$, which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if diam $(\overline{G}) \leq 3$.

G. Theorem

Let G be an r - dimensional Benes Network, $r > 2$, which is Super Strongly Perfect, then diam $(G) \geq 3$ if and only if $\gamma(\overline{G}) = 2$.

Proof:

Let G be a Benes Network which is Super Strongly Perfect

Since G is a bipartite graph, this theorem is proved by the theorem 5.5.

IX. CONCLUSION

We have given the characterization of Super Strongly Perfect graphs. We have analyzed the structure of Super Strongly Perfect Graph in Butterfly, Wrapped Butterfly and Benes Networks. We have presented the characterization of Super Strongly Perfect graphs in Butterfly, Wrapped Butterfly and Benes Networks. Also we have investigated the relationship between diameter, domination and co - domination numbers of Butterfly, Wrapped Butterfly and Benes Networks. This investigation can be applicable for the remaining well known architectures also.

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