DESIGN OPTIMIZATION OF TWO STAGE GEAR BOX USING SELECTIVE BREEDING ALGORITHM

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Abstract

An engineering design involves large numbers of design variables, and it is requires knowledge, experience, and judgment to specify these variables and design effective engineering systems. Design optimization is the application of numerical algorithms and techniques to engineering design to improving the system's performance, weight, reliability, and cost. An ideal gearbox is a typical and complex one, since it has fixed as well as sliding gears to transmit a controlled relative rotational motion between shafts. Since the functions of all mechanical members of an automatic machine tool are linked with its gear box, it should be strong enough to withstand the sudden variations in load. Hence there exists a need for designing an optimum gearbox for efficient performance. In this paper, a two stage gear box design is optimized by Evolutionary algorithms such as Selective Breeding Algorithm and Genetic Algorithm and optimized results are compared.

Keywords: Gearbox design, Multi objective optimization, Genetic Algorithm and Selective Breeding Algorithm

I. INTRODUCTION

Gearbox is the most commonly used mechanical element in various kinds of machines like metal cutting machine tools, automobiles, material handling equipments, rolling mills, marine power plants and aircrafts etc. A single speed gearbox has a gear pair with a constant speed ratio, contained in a separate casing and intended to reduce or increase the angular speed of the output shaft as compared with that of input shaft. In a multi-speed gearbox, the speed of the output shaft is varied in discrete steps; there may be increased or decreased speeds, from the input speed. To achieve the various speeds of the output shaft, number of intermediate shafts with gears are arranged kinematically between the input and output shafts with a gear shifting mechanism. Optimization plays an imperative role in various engineering applications. In engineering design action, a best possible design is achieved by comparing some alternative design solutions by using previous problem information. Optimization algorithms provide systematic and efficient ways of creating and comparing new design solutions in order to achieve an optimal design.

Senthil Kumar et al [1] evaluated an optimization of the asymmetric spur gear drive is carried out using an iterative procedure on the calculated maximum fillet stresses through FEM for different rack cutter shifts and finally the optimum values of rack cutter shifts are suggested for the given center distance and the speed ratio of an asymmetric gear drive. Hong Zhong Huang et al [2] developed developed a three-stage spur gear reduction unit with an objectives of minimizing the volume and maximizing the surface fatigue life is used to illustrate the effectiveness interactive physical programming approach. H. U. Nwosu and A. U. Iwuoha [3] analysed the failures of gearbox units after about 10,000 service hours when the design service life of 45,000 hours was expected was performed to establish the cause of failure. The gear tooth characteristics, namely, beam strength, maximum allowable dynamic load, allowable static load and limiting tooth wear load were analysed for ability to cope with the duty load. The failed gear components were examined visually and metallurgically. Yuji Ohue et al [4] discussed the fatigue test employed for a steel gear pair was carried out using a power circulating gear testing machine. Furthermore, the dynamic characteristics of both steel and sintered gears were measured using a power circulating gear testing machine and were analyzed in a time-frequency domain by the continuous and discrete wavelet transforms. Remy Uche and Ukueje Etabiese Wisdom [5] made attempts to design a model with high reliability and capability of predicting gear life in functional machinery. The work, therefore, advocates the use of a model based on statistical methods for predicting the fatigue life of gear components in machinery.

Padmanabhan et al [6] made an attempt to optimize bevel gear pair design using a non-linear programming optimizing software LINGO and meta-heuristics. The efficiency of the proposed algorithms is validated through gear design problems and the comparative results were studied. Ya Feng Li et al [7] According to the performance requirements of three-stage wind turbine gear box, the mathematical model was established as object function, design of the gear box was optimized with genetic algorithm and the examples showed that design of the gear based on genetic algorithm can gain more optimized result. Bernd-Robert Hohn et al [8], different methods were discussed for power loss reduction in a gearbox. The challenge is substantial power loss reduction with only minor impact on load carrying capacity, component size and weight and noise generation. Adequate compromises have to be proposed. Sriramya et al [9] proposed a novel evolutionary algorithm based on selective breeding algorithm for the Bin Packing application and optimal results were compared with other packing algorithms. Padmanabhan et al [10] developed a Real Coded Genetic Algorithm to optimize the design of Bevel gear pair and a combined objective function with maximizes the Power, Efficiency and minimizes the overall Weight, Centre distance.

The gearbox problem consists of determining the design variable such as module, gear thickness and number of teeth in order to optimize the design. A number of objective functions by which optimality of gearbox design are include: 1. Maximization of Power transmitted, 2. Minimization of Weight, 3. Maximization of Efficiency and 4. Minimization of Center distance. Several design conditions should be considered in the design of gearboxes: bending stress, crushing stress, shear stress and center distance etc.

II. PROBLEM DESCRIPTION

The problem involves the determination of optimum values for all the decision variables such as the number of teeth, thickness, module, and power so as to increase the power output and efficiency and reduce the volume and center distance of gearbox. The feasibility of a design solution depends on satisfaction of a number of equality and inequality constraints. The resulting optimization problem becomes more complicated due to the presence of multiple conflicting objectives. The preset work is carried out to increase the power output and to reduce the volume of the gearbox. Because the size and the power delivered are vital in the case of automobile or machine tool as it may improve its performance.

A Two stage Gear box problem was adopted to test the effectiveness of the new Evolutionary algorithm, Selective Breeding Algorithm in this paper. The test problem as follows: A gearbox is required to transmit 18 kW from a shaft rotating at 2650 rpm. The desired output speed is approximately 12 000 rpm. For space limitation and standardization reasons a double step-up gearbox is requested with equal ratios [11].

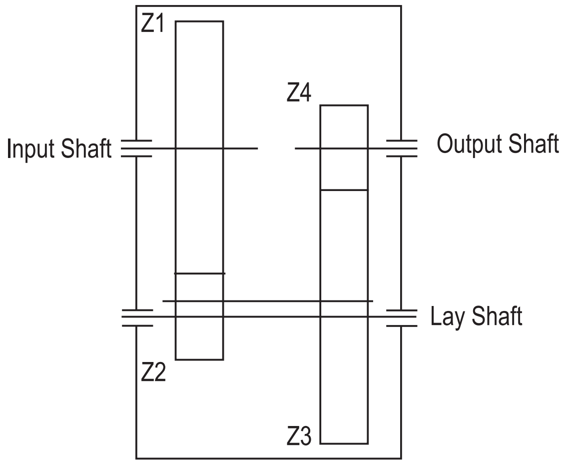

Fig. 1. Layout of Two Stage Gearbox

A. Assumptions

The various assumptions involved in the work are described below;

- All the gears in the gearbox are spur gears.
- The thicknesses of the gears are same in a gear-pair.
- In a gear-pair, wheel is assigned the larger number of teeth between the mating gears.

Table 1. Gearbox Details

Gears	No. of Teeth	Thickness (mm)
Z1	38	23
Z2	18	23
Z3	38	18.6
Z4	18	18.6

B. Objectives

Objective function is a function formed with the decision variables or design variables, whose values determine the solution to the problem. The objective functions are listed below were adopted from [12].

- Maximization of power delivered by the gearbox.
- Minimization of overall gear material used, which is directly related to the weight and cost of the gearbox.
- Maximization of efficiency of the gearbox.
- Minimization of center distance between input and output shafts

Equations (1), (2), (3) and (7) represent the above said objective functions.,

$$\text{Maximize, } f_1 = P, \text{ Where } P^{(L)} \le P \le P^{(U)} \tag{1}$$

$$\text{Minimize, } f_2 = \frac{\pi}{4} \sum_{i=1}^{3} m^2 (Z_{\pi}^2 + Z_{wi}^2)\, b_i \text{ for } i = 1, 2, \tag{2}$$

$$\text{Maximize, } f_3 = 100 - P_L \tag{3}$$

$$P_L = \frac{50f}{\cos\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \tag{4}$$

Where, $f = 0.08$, $\Phi = 20°$

$$H_t = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_0}{r} \right]^2 - \cos^2\Phi \right)} - \sin\Phi \tag{5}$$

$$H_s = (i+1) \times \sqrt{\left(\left[\frac{R_0}{R} \right]^2 - \cos^2\Phi \right)} - \sin\Phi \tag{6}$$

$$\text{Minimize, } f_4 = \frac{m}{2} (Z_1 + Z_2) \tag{7}$$

The power loss 'P_L' is calculated for all gear pairs, added together and subtracted from 100 to get the efficiency of the gearbox [13].

C. Constraints

For the gearbox layout shown in Figure. 1 there are two gear-pairs, or $G = 2$. There also exist a number of constraints associated with this problem. By considering fixed number of teeth Z_i varying thickness values b_i and power delivered P, each gear must satisfy two constraints mentioned by eqn. (8) and (9) were adopted from [12].

$$\text{Bending stress: } \sigma_{bi} \le [\sigma_{bi}]_{al} \tag{8}$$

$$\text{Crushing stress: } \sigma_{ci} \le [\sigma_{ci}]_{al} \tag{9}$$

Where, $\sigma_b =$ bending stress

$\sigma_c =$ crushing stress

$[\sigma_b]_{al} =$ Allowable bending stress

$[\sigma_c]_{al} =$ Allowable crushing stress

The bending and crushing stresses developed in the ith gear-pair are calculated by the eqns. (10) and (11).

Bending stress developed in the ith gear-pair:

$$\sigma_{bi} = \frac{97500 P K_c K_d (r_i + 1)}{w_i a_i b_i m r_i y_i \cos\beta} \tag{10}$$

Crushing stress developed in the ith gear-pair:

$$\sigma_{ci} = \frac{0.59\,(r_i + 1)}{r_i a_i} \sqrt{\frac{97500\, P K_C K_d (r_i + 1)\, E}{w_i b_i \sin 2\beta}} \tag{11}$$

Where,

k_c = 1.5 (Stress concentration factor)

k_d = 1.1 (Dynamic load factor)

r_i = Transmission ratio

w_i = Speed of the wheel in rpm

a_i = Centre distance for the corresponding gear-pair in mm

b_i = Thickness of the gear-pair in mm

y_i = Form factor

β = Pressure angle (20°)

$E =$ Young's modulus of the gear material in N/mm^2

The transmission ratio is defined as the ratio of the number of teeth Z_{wi} in wheel to the number of teeth Z_{pi} in pinion and it is expressed by eqn. (12).

$$r_i = \frac{Z_{wi}}{Z_{\pi}} \tag{12}$$

The center distance for the corresponding gear-pair is found by eqn. (13).

$$a_i = \frac{m\,(Z_{wi} + Z_{\pi})}{2} \tag{13}$$

$$y_i = 0.52\left(1 + \frac{20}{Z_{wi}}\right) \tag{14}$$

Form factor y_i is calculated by the eqn. (14),

The thickness and power value should lie between lower and upper limit values, these constraints are shown by the eqns. (15) and (16).

$$b^{(L)} \le b_i \le b^{(U)},\ \text{For } i = 1, 2. \tag{15}$$

$$p^{(L)} \le p \le p^{(U)} \tag{16}$$

With the number of teeth in gears are also kept as decision variables, the resulting problem must involve additional constraints considering the following aspects.

- The center distance between two shafts must be maintained by all corresponding gear-pairs as shown in eqn. (17)

For center distance:

$$Z_1+Z_2 = Z_3+Z_4 \tag{17}$$

(between Input shaft and Output shaft)

- The maximum gear-ratio in any gear-pair must not exceed a limit r^{max} eqn. (18)
- For maximum permissible gear ratio in all gear-pairs:

$$\frac{Z_{wi}}{Z_{\pi}}\ \text{for } i = 1, 2 \tag{18}$$

- Moreover, the nature of these additional decision variables (number of teeth) requires them to be treated as non-zero positive intergers.eqn. (19) represents this constraint.

Number of teeth to be integers,

$$Z_i\ \varepsilon\ I,\ \text{for } i = 1, 2. \tag{19}$$

Number of teeth in each gear pair:

$$Z_{wi} \ge Z^{(L)}\ \text{for } i = 1, 2. \tag{20}$$

$$Z_{\pi} \ge Z^{(L)}\ \text{for } i = 1, 2. \tag{21}$$

a. Variable elimination

We can use the linear equality constraints to eliminate two decision variables.

$Z_1+Z_2 = Z_3+Z_4$ (between Input shaft and Output shaft)

For example, equation (22) can be used to eliminate two variables as shown below.

$$Z_4 = Z_1 + Z_2 - Z_3 \tag{22}$$

D. Normalized Weighted Multiple-Criterion Objective Function:

The multiple-criterion objective function is capable of incorporating the evaluation values of different criteria together. The quality of the design can be evaluated by four different criteria:, maximization of power, minimization of volume of material, maximization of efficiency and minimization of center distance. Since these criteria are on different scales, to reflect their real contribution to the multiple-criterion objective function their values have to be normalized to the same scale. Hence the combined objective function adopted in the previous chapters is used here also.

$$\mathrm{COF} = \left[\left(\frac{\text{power}}{\text{max} \cdot \text{power}} \times \mathrm{NW}_1\right) + \left(\frac{\text{min} \cdot \text{weight}}{\text{weight}} \times \mathrm{NW}_2\right) + \left(\frac{\text{efficiency}}{\text{max} \cdot \text{efficiency}} \times \mathrm{NW}_3\right) + \left(\frac{\text{min} \cdot \text{centdist}}{\text{centdist}} \times \mathrm{NW}_4\right)\right] \tag{23}$$

Where NW_1, NW_2, NW_3 and $NW_4 = 0.25$

III. METHODOLOGY FOR GEAR BOX OPTIMIZATION

After the widespread literature review, many researchers have shown more interests on non-traditional optimization techniques such as Genetic Algorithm, Simulated Annealing and artificial immune system etc and applied the same in various engineering fields [14, 15]. Here, an attempt to be made with a new Selective Breeding Algorithm (SBA) and Genetic

Algorithm for the optimum design of two stage gear box.

A. Genetic Algorithm

In this work one type of Genetic Algorithm, Real Coded Genetic Algorithm (RGCA) is used. The RCGA uses Mixed Integer Representation for representing the control variables, Tournament selection, Crossover and Polynomial Mutation.

a. Representation

Mixed integer representation is used for the control variables. The module, thickness, number of teeth in pinion, the power, maximum power, minimum weight, maximum efficiency, minimum center distance and COF are represented in a control string. Module, thickness of gear pair and power are represented as continuous variables within limits. The number of teeth is represented as discrete variable.

b. Tournament Selection

The tournament selection provides a selective pressure by holding a tournament competition among individuals. The best individual from this group is selected as parent. That is, any two strings are randomly selected from this population and the COF value is compared. The string having the lowest COF will be stored in the new mating pool.

c. Crossover

The simulated binary crossover performs the crossover variable-wise using Simulated Binary Crossover (SBX) operator. It creates children solutions in proportion to the difference in parent solutions. Simple reproduction can also occur which replicates a single individual into the new population.

d. Polynomial Mutation

Newly generated offspring undergo polynomial mutation. Like in the SBX operator, the probability distribution can also be a polynomial function, instead of a normal distribution. This randomly modifies the genes of an individual subject to a small mutation factor, introducing further randomness into the population.

e. Stopping criteria

The approach followed in this work, is to stop the computation after reaching the required number of iterations. The maximum number of iterations adopted here is 10. For each iteration the population is generated continuously by 200 times.

B. Selective Breeding Algorithm

Selective breeding is the procedure of breeding plants and animals for exacting character. normally, strains that are selectively bred are cultivated, and the breeding is sometimes done by a expert breeder. Bred animals are known as breeds, while bred plants are known as varieties, cultigens, or cultivars. The cross of

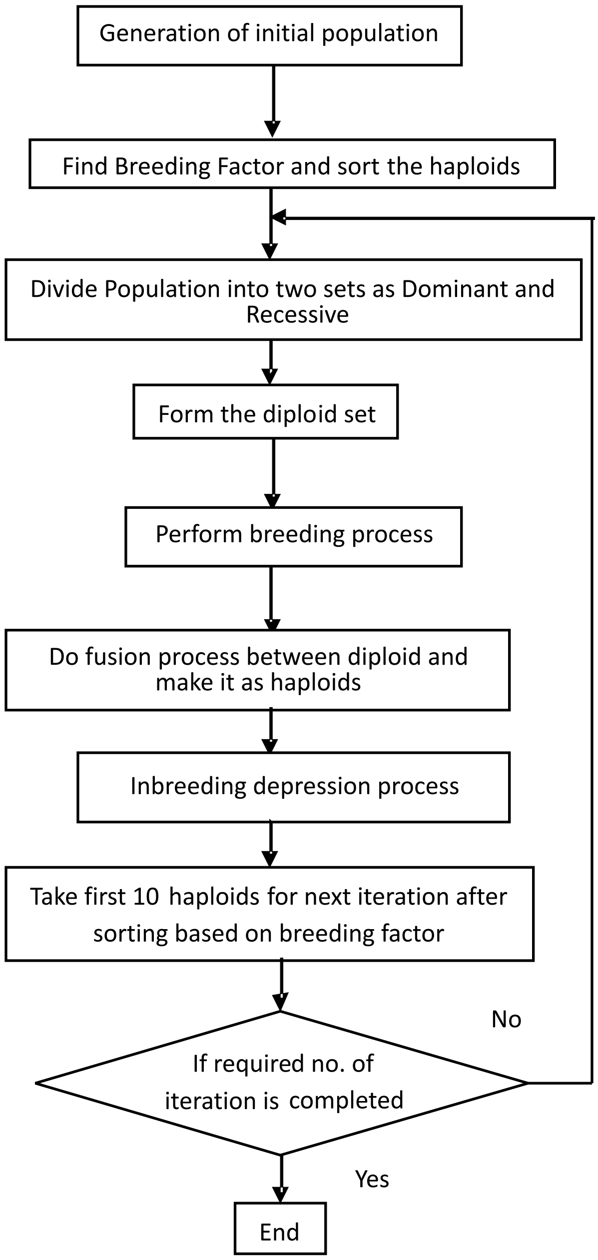

Fig. 2. Flow chart for Selective Breeding Algorithm [9]

animals result is called a crossbreed, and crossbred plants are called hybrids. The term selective breeding is synonymous with artificial selection [16]. The new selective breeding algorithm performs step by step is show in the flow chart.

IV. RESULTS AND DISCUSSION

The parameters considered for the multi speed gear box design is given below.

Table 2. The Input Values of Gear Box with their Bounds

Parameter/Constraint		Values for Spur Gear Pair
Gear material		655M13 Case Hardened Steel
Gear Thickness	Lower Limit	10 mm
	Upper Limit	30 mm
Power delivered	Minimum	18 kW
	Maximum	20 kW
Max Speed ratio		4.45
Number of Teeth	Lower Limit	14
	Upper Limit	38
Permissible bending strength		345 N/mm^2
Permissible crushing stress		1080 N/mm^2
Young's modulus		2.1×10^5 N/mm^2
Input and Output speed		2650 rpm and Approx 12000 rpm
Module	Minimum	1
	Maximum	3

The test problem was solved in two cases. Case 1 represents module is kept constant and other variables like power, thickness, teeth and center distance is varied. While in Case 2, all variables are varied to get the optimized results.

A. Constant Module (Case I)

In this case the module is kept constant (module = 2 mm). The variables are Power, Gear Thickness, Number of teeth and Center distance are varied. The optimized results are tabulated in Table 3.

B. Varying all the considered design variables (Case II)

The problem can be more flexible by allowing some additional parameters as decision variable in order to get a better insight into the gearbox design problem. In this case all the parameters such as power, thickness, number of teeth, module and center distance are considered as variables. The module of the gear train is allowed to vary in the range [1, 1.125, 1.25, 1.375, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3]. This way a discrete set of 11 different modules are considered. For real-parameter decision variables, identical operators and parameters as those used in the previous case are adopted here.

From the table 4 and 5 , it clearly shows that, large volume of gear box is reduced by selective breeding algorithm and the same time power also considerably increased from the existing design. Selective breeding algorithm performed better than Genetic algorithm and existing design in overall comparison.

V. CONCLUSION

In this work, a two stage gear box was taken and optimized results were obtained from selective

Table 3. Optimum results for Constant Module (Case I)

Design Tool	No. of Teeth		Gear Thickness		Power (kW)	Volume (mm^3)	Eff. (%)	Center Dist. (mm)
Existing design	Z_1=38	Z_2=18	b_1=23	b_2=23	18	231060	97.17	56
	Z_3=38	Z_4=18	b_3=18.6	b_4=18.6				
GA	Z_1=38	Z_2=18	b_1=21.4		18.02	218840.6	97.17	56
	Z_3=38	Z_4=18	b_3=18					
SBA	Z_1=34	Z_2=16	b_1=22		18.11	177437	96.88	50
	Z_3=34	Z_4=16	b_3=18					

Table 4. Optimum results obtained for GA (Case II)

	Gear-pair no, i		Module (mm)	Power (kW)	Volume (mm^3)	Eff. (%)	Center dist. (mm)
	1	2					
Z_{pi}	18	18	1.5	18	156215.6	97.17	56
Z_{wi}	38	38					
b_i	27	23					
Z_{pi}	18	18	2	18.02	218840.6	97.17	56
Z_{wi}	38	38					
b_i	21.4	18					

Table 5: Optimum results obtained for SBA (Case II)

	Gear-pair no, i		Module (mm)	Power (kW)	Volume (mm^3)	Eff. (%)	Center dist. (mm)
	1	2					
Z_{pi}	16	16	1.5	18.08	120269	96.88	50
Z_{wi}	34	34					
b_i	25.8	22.4					
Z_{pi}	16	16	2	18.11	177437	96.88	50
Z_{wi}	34	34					
b_i	22	18					

breeding algorithm and genetic algorithm. Evolutionary algorithms shows the considerable reduction in volume of gear box, which is directly related to the weight and cost of the gearbox. Weight reduction reduces the amount of material consumed in the development the component in an industry. The material use is directly proportional to the cost. In the designing of machine tool and automobile gearboxes takes the benefit of evolutionary, the manufacturers can assertively face the viable environment of producing the gearboxes at reasonable cost. As a future work, the Selective breeding algorithm can be evaluated by optimizing various design application like bearing, simple gear train and epicyclic gear train etc.

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