SIMULATION OF GEOMETRICAL CROSS-SECTION FOR PRACTICAL PURPOSES

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ABSTRACT

In the present work we have tried to optimize the cross sectional area of curved beam, by properly selecting the parameters with the help of computer. Flexural formula is reasonably held good for this calculation [1]. This study forces on both bending stresses either tensile or compressive along with displacement centroidal axis with respect to neutral axis called eccentricity. Software's developed in the present work have been run successfully; trapezoidal section is the most suitable for curved beam among circular, rectangular, triangular trapezoidal T and I - sections. Trapezoidal section takes maximum total stresses with minimum shift in eccentricity.

Keywords: Curved beam, flexural equation, eccentricity, bending stresses, trapezoidal cross-section.

I. INTRODUCTION

Machine members and structures subjected to bending are not always straight as in the case of crane hooks, chain links etc., before a bending moment is applied to them. For initially straight beams the simple bending formula is applicable and the neutral axis coincides with the centroidal axis [4]. A simple flexural formula may be used for curved beams for which the radius of curvature is more than five times the beam depth. For deeply curved beams, the neutral and centroidal axes are no longer coincide and the simple bending formula is not applicable.

The total deformations of the fibers in curved beams are proportional to the distances of the fibers from the neutral surfaces, the strains of the fibers are not proportional to these distances because the fibers are not of equal length; where as in straight beam the fibers are of equal length and fibers are of equal length and hence the strains in a straight beam, as well as total deformations are proportional to the distances of fibers from neutral axis [2]. But for bending stresses that do not exceed the elastic strength of the material the stress on any fiber in the beam is proportional to the strain of the fiber, and hence the elastic stresses in the fibers of curved beam are not proportional to the distances from the neutral surface it follows, then that the resisting moment in a curved beam is not given the expression $\sigma\ I/C$. Since this expression is obtained on the assumption that stress varies directly as the distance from the neutral axis. For the same reason, neutral axis in a curved beam does not pass through the centroid of section [2].

II. ASSUMPTIONS TAKEN IN PRESENT ANALYSIS [2, 3]

- Material of the beam is homogeneous and isotropic.
- The material of the bar is stressed within the elastic limit and thus Obeys Hook's law.
- The values of the Young's modulus are the same for beam material in tension as well as compression.
- A transverse section of the beam, which is plane before bending, will remain plane after bending.
- The resultant pull or thrust on a transverse section of beam is zero.
- The transverse cross-section has at least one axis of symmetry, and bending moment lies on this plane.

III. ANALYSIS

The distribution of stresses in a curved beam is

$$\sigma = \frac{My}{Ae\,(R - y)} \qquad ...(1)$$

An optimum section of a curved beam can be designed by relating the inner and outer fiber stresses in the same ratio as the tensile and compressive strength of beam material is steel, the and compressive strength are nearly equal.

Expressions of '*e*' from above equation are different for different sections. In most of the engineering problems, the magnitude of *e* is very small and it should be calculated precisely to avoid large percentage error in the final results [1].

IV. STRESS DISTRIBUTION EXPRESSION FOR CURVED BEAMS

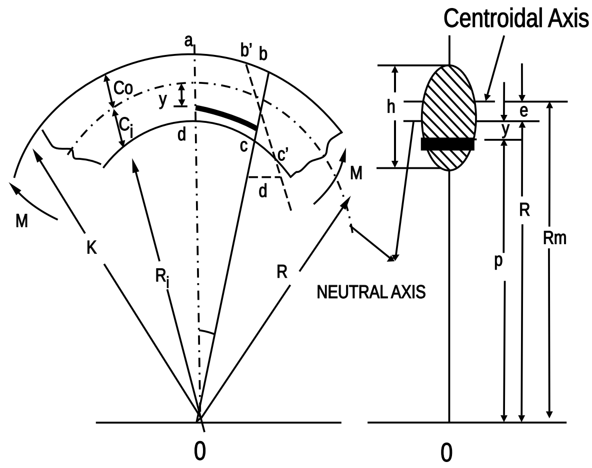

Fig. 1. Curved beam subjected to bending.

A curved beam subjected to bending moment, M is shown in Figure 1. An element a,b,c,d is defined by the angle θ. Bending moment M causes section b,c to rotate through d to b'c'.

According to [5] the strain on any fiber at a distance p from center O is:

$$\varepsilon = \frac{(R-p)\,d\theta}{p\theta} \qquad ...(2)$$

The moment stresses corresponding to this strain is

$$sog,a = \varepsilon\, E = \frac{E\,(R-p)\,d\theta}{p\theta} \qquad ...(3)$$

Since there are no axial external forces acting on the beam, the sum of the normal forces acting on the section must be zero [5]. Therefore

$$\int \sigma\, dA = \frac{ED\theta}{\theta} \int \frac{(R-p)\,dA}{p} = 0 \qquad (4)$$

We arranging equation (3) in the from

$$\frac{Ed\theta}{\theta}\left[R \int \frac{dA}{p} - \int dA \right]$$

Solving the expression in parenthesis. This gives.

$$R \int \frac{dA}{A} - A = 0$$

$$R = \frac{A}{\int dA/p} \qquad ...(5)$$

This important equation is used to find the location of neutral axis with respect to center of the curvature o of the cross-section [5,9] .The equation indicates neutral axis and centroidal are not coincident and the neutral axis shifted towards center of curvature, it is located between the centroidal axis and center of curvature.

Now balancing the external applied moment against the internal resisting moment

$$\int (R-p)\,(\sigma\, dA) = \frac{Ed\theta}{\theta} \int \frac{(R-p)^2}{p}\, dA = M \qquad ...(6)$$

$$(R-p)^2 = R^2 - 2Rp + p^2$$

Equation 6 can be written as

$$M = \frac{Ed\theta}{\theta}\left[R^2 \int \frac{dA}{p} \int dA - R\int dA + \int pdA \right] \qquad ...(7)$$

now that R is a constant, comparing the first two terms in parentheses with equation(4). These terms vanish and we have left with

$$M = \frac{Ed\theta}{\theta}\left[-R \int dA + \int pdA \right] \qquad ...(8)$$

The first integral in this expression is the area A and second integral is area R_mA. .

Therefore,

$$M = \frac{Ed\theta}{\theta}\,(R_m - R)\,A \qquad ...(9)$$

Since $e = R_m - R$

$$M = \frac{Ed\theta}{\theta}\, eA \qquad ...(10)$$

Now using equation (2) once more and arranging finally we obtain.

$$\sigma = \frac{my}{Ae\,(R-y)} \qquad ...(11)$$

According to [6] a modified equation for computing the stress at extreme fiber of curved beam is given by

$$\sigma = \frac{kMh}{1}$$

$$\text{in which, } K_i = \frac{\frac{M}{Ae}\frac{(h_i - e)}{r_i}}{\frac{2Mh_i}{2I}}$$

$$K_o = \frac{\frac{M}{Ae}\frac{(h_i + e)}{r}}{\frac{2\,Mh_o}{2I}}$$

values of the correction factors K_i for the inside fiber and K_o for outside fiber for some of the common sections with different degrees of curvature. Value of K can also be read from graph from [6].

V. FLOW CHART AND COMPUTER PROGRAMS:

A. Program Development [8]

The flow chart for software for design optimization of curved beam with trapezoidal the program developed is shown in figure 2.

B. Post Processor

The eccentricity is obtained with the help of graph between total tensile and compressive stresses using software.

The program for stress distribution analysis of curved beam used in crane hook will give total stress at the desired fiber (top and bottom fiber).

Table 1. Input Data

1	Load on beam	1000 kg, 200 kg
2	Inner radius of curved beam	1 cm
3	Depth of section	2 cm
4	Length of straight portion ofbeam	4 cm
5	Allowable bending stress	4410 Kg/cm^2
6	Allowable shear stress	4310 Kg/cm^2
7	Modulus of elasticity	$2.07*10^6$ Kg/cm^2
8	Modulus of rigidity	$0.89*10^6$ Kg/cm^2

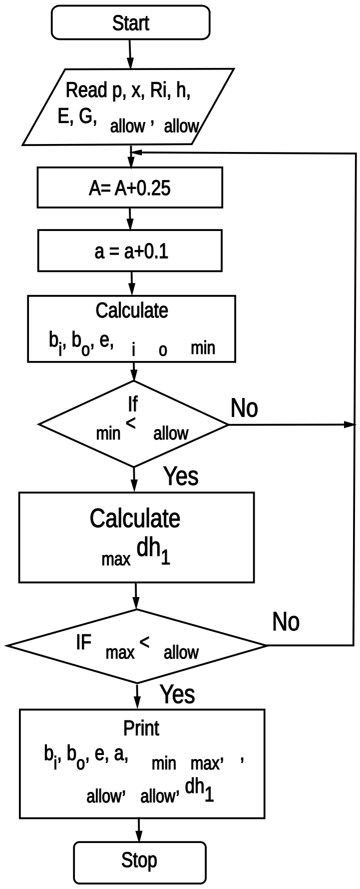

Fig. 2. Design optimizaiton of curved beam

Following output parameters will be obtained for optimal design of curved beam with trapezoidal section.

(i) Area of cross section

(ii) Inner and outer width of the trapezoidal section

(iii) Distance between neutral and centroidal axis

(iv) Maximum bending stress

(v) Maximum shear stress

(vi) Distance of fiber at which shear stress is maximum from top fiber.

VI. CASE STUDY

for eccentricity of curved beam and stress distribution analysis of curved beam used in crane hook are given with results obtained by the computation.

Input data for calculation is shown in table 1 for alloy steel.

VII. RESULTS AND DISCUSSION

Results obtained from the software for stress distribution analysis of curved beam show that stress can be easily calculated on any fiber of beam section if all the dimensions of section, load and distance from centroidal axis on it are known to us.

Eccentricity is obtained with the help of graph of curved beam in table (2) using load P.

$P = 1000, 2000, 3000$ kg.

$A = 19.64, 30, 15, 22.5, 18, 15$ cm^2 respectively.

I_{xx} = 30.68, 62.5, 20.83, 45.1, 53.50, 51.52 cm^4 respectively.

$Y =$ distance top and bottom fiber form centroidal axis is different for different sections.

Eccentricity has been calculated with the help of graph of circular, rectangular, triangular, trapezoidal, *T* and *I* – sections with maximum possible load is

I $-0.65, -0.8, +0.5, -0.25, +0.5, -1.3$cm

II $-0.60, -0.80, +0.6, -0.3, +0.4, -1.4.$ cm

Total stress has been calculated at the outermost fibers of circular, rectangular, trapezoidal *T* and *I* – sections is

I – 504.56, 257.88, 322.60, 181.11, 272.46, 368.98 kg/cm^2 respectively.

II – 1009.12, 515.76, 645.20, 362.22, 544.92, 737.96 kg/cm^2 respectively.

Table 2. Stress distribution Analysis of Curved beam

Sections	Load (Kg)	Area (cm)^2	Distance from neutral axis (R) (cm)	Distancefrom Centroidal Axis (cm)		Bending Moment (kg-cm)		Bending Stress (kg/cm^2)		Total Stress (kg/cm^2)	
				Top Fiber Y_1	Bottom Fiber Y_2	Horizontal	Vertical	Tensile	Compressive	Tensile	Compressive
Circular	1000	19.64	6.77	+2.5	−2.5	6062.18	3500	453.64	208.96	504.56	158.04
	2000					12124.36	7000	907.28	417.92	1009.12	316.08
	3000					18186.54	10500	1360.92	626.88	1513.68	474.12
Rectangular	1000	30	6.69	+2.5	−2.5	6062.18	3500	224.55	102.38	257.88	69.05
	2000					12124.36	7000	449.10	204.76	515.76	138.10
	3000					18186.54	10500	673.65	307.14	773.64	207.15
Triangular	1000	30	6.69	+2.5	−2.5	6062.18	3500	224.55	102.38	257.88	69.06
	2000					12124.36	7000	449.10	204.76	515.76	138.10
	3000					18186.54	10500	673.55	307.14	773.64	207.15
Trapezoidal	1000	22.5	2.17	+2.78	−2.22	6062.18	3500	136.67	787.89	181.11	743.45
	2000					12124.36	7000	273.34	1575.782	362.22	1486.902
	3000					18186.54	10500	410.01	363.67	543.33	230.35
T-Section	1000	18	2.76	+3.50	−1.50	6062.18	3500	216.90	16.15	272.46	39.41
	2000					12124.36	7000	433.80	32.30	544.92	78.96
	3000					18186.54	10500	650.70	48.45	817.38	118.23
I-Section	1000	15	6.52	+2.50	−2.50	6062.18	3500	302.31	134.73	368.98	68.06
	2000					12124.36	7000	604.63	269.46	737.96	136.12
	3000					18186.54	10500	906.93	404.19	1106.94	204.18

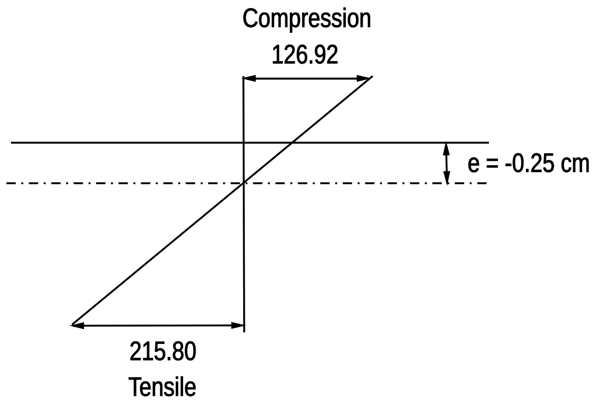

Fig. 3. Eccentricity of Trapezoidal section of curved beam

It can be concluded from above results that trapezoidal section is the most suitable section for the curved beam considered because minimum eccentricity and maximum stress is the lowest with this section shown in figure 3.

Same conclusions are made on changing the magnitudes P, A, and Ixx, as it is evident from the results.

Software for design optimization or curved beam with trapezoidal section has been run for sample problem.

Results obtained are;

A =1.75 cm^2

b_i = 1.46 cm

b_o = 0.29 cm

Maximum bending stress = 3898.51 kg/cm^2

Maximum shear stress = 236.97 kg/cm^2

Maximum deflection = 0.016 cm

Thus beam is safe in both bending and shear.

VIII. CONCLUSIONS

Following general conclusions can be made from the results obtained.

(i) The range of the stress distribution analysis from top to bottom and vice versa of curved beam can be done easily by the software developed by us. Thereby saving human labour and the time of the designer.

(ii) Trapezoidal section is the most suitable section for curved beam used in crane hooks, clamps etc., Among circular, rectangular, triangular trapezoidal T and $I-$ sections because neutral axis minimum shifted towards centroidal axis.

(iii) Trapezoidal section will take maximum bending Stress. So, best optimum section is trapezoidal section for design criteria.

Nomenclature

$A =$ Area of Cross-Section, cm^2

$C =$ Inner radius of Crane Hook, cm

$E =$ Modulus of Elasticity, kg/cm^2

$G =$ Modulus of Rigidity, kg/cm^2

$I =$ Moment of Inertia, cm^4

$M =$ Bending Moment, kg-cm

$N =$ Normal Force, kg_f

$P =$ Applied Force, kg_f

$R =$ Radius of Neutral Axis, cm

R_i = Radius of Inner Fiber, cm

R_m = Radius of Centroidal Axis, cm

R_o = Radius of Outer Fiber, cm

$U =$ Strain Energy, kg-cm

b_i = Width of Inner Fiber, cm

b_o = Width of outer Fiber, cm

$d =$ Diameter of Circular Section, cm

$\alpha =$ Ratio of Inner Width to Outer Width

$C** =$ Linear Strain

$\partial =$ Deflection, cm

$\sigma =$ Bending Stress, kg/cm^2

$\sigma_t =$ Total Stress, kg/cm^2

$\tau =$ Shear Stress, kg/cm^2

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